CDOs and the Financial Crisis: Credit Ratings and Fair Premia

Marcin Wojtowicz*

VU University Amsterdam (FEWEB), De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands
Tinbergen Institute Amsterdam, Roetersstraat 31, 1018 WB Amsterdam, The Netherlands

Abstract

In this paper we use the market-standard Gaussian copula model to show that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds. Our findings imply that credit ratings are not sufficient for pricing, which is surprising given their central role in structured finance markets. The yield enhancement on tranches is attributed to a concentration of risk premia. This illustrates limitations of the rating methodologies being solely based on estimates of real-world payoff prospects. We further show that payoff prospects as well as credit ratings of CDO tranches have low stability. If credit conditions deteriorate, then prices and ratings of CDO tranches are likely to decline significantly more than prices and ratings of corporate bonds. Default contagion exacerbates the pace and severity of tranche re-pricing and downgrading.

Keywords: Collateralized debt obligations, Credit ratings, Fair premia, Structured finance, Rating agencies
JEL classification: C52, G01, G11

1. Introduction

There is no doubt that structured credit products have only developed so rapidly because they offered higher coupons relative to equally-rated corporate bonds. Yield enhancement on structured securities was particularly appealing to investors who assumed that credit ratings represent a universal and robust indication of payoff prospects across different asset classes. For example, the premise that the

*phone: +31 20 5983821
Email address: `mwojtowicz@feweb.vu.nl` (Marcin Wojtowicz)

highest rating grade is a guarantee of very low default risks has encouraged many institutional investors to add triple-A securitized tranches to their portfolios. The frailty of such a rating-based approach has only become evident in 2007-2008 when the mounting losses associated with subprime mortgages eventually led to the collapse of the structured finance markets.

The main result in this paper is that CDO-structuring concentrates risk premia in spreads of non-equity tranches, which provides a clear-cut explanation for the tranche yield enhancement. Finance theory indicates that credit ratings, which measure only pure default risk, cannot fully account for fair premia due to risk aversion of investors. Strictly speaking, credit ratings are based on expected losses or default probabilities calculated under the physical measure. In contrast, fair premia are closely related to expected losses calculated under the risk-neutral measure that is derived from (significantly) higher market-implied default probabilities. We show that CDO structuring results in high sensitivity of expected tranche losses to default probabilities of the underlying bonds. CDO tranches can therefore be tailored to have low real-world expected losses, while having much higher risk-neutral expected losses. For this reason CDO tranches can qualify for high credit ratings and offer significant yield enhancement relative to similarly-rated bonds.

In modeling credit ratings and fair premia of CDO tranches, we rely on the market standard one-factor Gaussian copula model. As a stylized example, we consider a portfolio of hundred BBB- bonds[1] with a market spread of 111.95 bps that decomposes into 53.06 bps of pure default risk compensation and 58.89 bps of risk premium. By securitizing this bond portfolio, we create a mezzanine CDO tranche of the same BBB- credit quality and hence roughly the same spread to compensate for pure default risk, but with a much higher total fair spread of 320.69 bps. This is an almost three-fold increase in the total spread due to a five-fold multiplication of the risk premia of the underlying bonds, which illustrates that tranche spreads cannot be derived on the basis of similarly-rated bonds. We further re-securitize the BBB- tranches to create a CDO-squared[2] with even higher spreads. For example, under realistic assumptions, we construct a CDO-squared tranche with a BBB- rating and a fair spread of 795.71 bps.

Our results on the tranche yield enhancement demonstrate that the current rat-

[1]Whenever we discuss a credit rating without indicating the rating agency, e.g. a BBB- bond, we always refer to the S&P rating.

[2]A CDO-squared is a CDO-type security backed by a collateral pool consisting of tranches from other CDO deals.

ing system can be gamed if it is used for pricing purposes. Producing CDOs allows for boosting premia on highly rated securities. This creates vast possibilities for rating arbitrage, which made the structured finance industry so profitable. The excess tranche spreads can be distributed between CDO investors and issuers. The investors are able to increase their returns on highly rated portfolios, while the issuers are compensated for their efforts and risks associated with originating and structuring CDOs. These results complement the study by Brennan et al. (2009) who propose an analytical model based on the CAPM and the Merton model to analyze the gains of an investment banker selling CDO tranches at the spreads of equally-rated corporate bonds.

Finally, we discuss reasons why structured securities are likely to perform poorly during unfavorable market conditions. The key to this analysis lies in high sensitivity of expected tranche payoffs to default probabilities of the underlying bonds. We show that an increase in default probability estimates of the underlying bonds, which is typical for a deterioration in credit conditions, has a much stronger effect on ratings and prices of CDO tranches than of corporate bonds. Moreover, default contagion is a crucial factor exacerbating the pace and severity of changes for CDO tranches.

The rest of this paper is organized as follows. Section 2 discusses the background of the structured finance markets. Section 3 explains the modeling approach and assumptions. In Section 4 we present our findings on the CDO yield enhancement and in Section 5 we analyze the sensitivity of tranche payoffs. In Section 6 we discuss the stability of tranche ratings and prices. Section 7 concludes.

2. Background

Structured finance transforms lower quality assets into securitized tranches that are better suited for investors' risk appetite. Producing structured securities begins with pooling (collateral) assets into large well-diversified portfolios, which allows for a substantial reduction of idiosyncratic risks. Subsequent prioritization of cash flows associated with each underlying portfolio creates several securities (tranches) of varying credit quality. Tranche investors bear the credit losses incurred by the underlying portfolio within pre-agreed limits and in return they receive premium payments. Most of the credit risk is concentrated in the first-loss (equity) tranche. More senior tranches are characterized by higher credit quality compared to the average quality of the collateral pool. In practice, highly rated AA and AAA tranches constitute about 60% of the volume of securitized portfolios rated by Fitch (2007). The ability of structured finance to produce such large volumes of highly rated tranches was particularly successful to meet the large market demand for very safe securities originating from institutional investors such as pension funds or money-market funds.

Benmelech and Dlugosz (2009a) analyze the practice of rating CDOs[3] and they use the term 'alchemy' to describe the apparent disparity between the credit quality of CDO tranches and the credit quality of their underlying collaterals. Equally intriguing is that highly rated tranches offer a significant yield enhancement relative to similarly-rated bonds. For example, in the run-up to the 2007-2009 financial crisis, triple-A structured securities provided as much as 50 bps in case of CDO-squareds.[4] Such attractive coupons were not common for triple-A assets in the corporate bond universe. The originators of CDOs have attributed the tranche yield enhancement to the 'leveraging' and 'correlation risk' created by prioritizing tranche payoffs (ABC of CDOs, 2004). However, most likely the implications of these terms were not fully understood by investors. Crouhy et al. (2008) point out that "the argument could be made that as the yields on structured instruments exceeded those on equivalently rated corporations, the market knew they were not of the same credit and/or liquidity risk. But investors still misjudged the risk".

Our focus on credit ratings is motivated by their predominant importance in the structured finance markets. Credit ratings have been essential because the complexity of securitized products limited the ability of unsophisticated investors to conduct independent risk assessment (Crouhy et al., 2008). There is also a growing consensus in academic literature that investors relied heavily on credit ratings not only for risk management, but also to infer fair premia; for a discussion we refer to Brennan et al. (2009), Coval et al. (2009a), Crouhy et al. (2008), Krahnen and Wilde (2008) and Firla-Cuchra (2005). For example, Krahnen and Wilde (2008) point out that "Ratings are used almost universally by investors, bankers, supervisors, and regulators as the relevant risk metric. The familiarity of markets with these letter ratings has probably encouraged investors to add these instruments to their portfolios, and has helped to establish the market for various ABS [CDO] products in the first place".

The rating agencies have been ambiguous about the meaning of credit ratings.

[3]Benmelech and Dlugosz (2009a) focus on collateralized loan obligations (CLOs), which are CDOs backed by portfolios of loans.

[4]We compare the spreads on a few dozen CDO tranches rated by S&P in 2006 (from S&P Ratings Direct database).

On the one hand, they advertised credit ratings as "a uniform measure of credit quality globally and across all types of debt instruments" (S&P, 2007).[5] The same document further reads, "In other words, an 'AAA' rated corporate bond should exhibit the same degree of credit quality as an 'AAA' rated securitized debt issue". On the other hand, the agencies asserted that credit ratings are merely "opinions about a relative creditworthiness of a security" (S&P, 2009). Similarly, the rating agencies indicated that credit ratings are not sufficient for pricing, but they did not explain fundamental differences in risks between like-rated bonds and securitized assets. For illustration, an S&P document explaining the "meaning behind structured finance ratings" states: "We recognize that the global capital markets may not always price similarly rated debt types the same, all things being equal. This is also true when comparing different securitized issues. Such differences may be based on both credit and non-credit or market considerations, including perceived prepayment risks based on asset or structural characteristics; seller/servicer characteristics; the asset class' historical track record; the availability of historical performance data; and market liquidity considerations, including the depth of secondary markets in certain sectors or markets." (S&P, 2007).

Credit ratings are an assessment of a security's credit quality. In case of corporate bonds, the rating process depends heavily on qualitative as well as quantitative components. Bonds are categorized into a number of grades according to their relative payoff prospects. These rating grades are not meant to represent precise estimates of default probabilities. Actual default performance of bonds typically varies between years. For example, BBB bonds rated by S&P have an average annual default rate of 0.26% with a standard deviation of 0.27% (based on 1985-2009 period), see S&P (2010). The rating agencies also publish cumulative default probabilities of bonds, which are more stable than annual default rates. For example, triple-A bonds rated by S&P have a 10-year historical default probability of 0.36% (S&P, 2005). Such statistics give investors an intuitive meaning to the 'relative ranking of payoff prospects' implied by credit ratings.

The rating methodologies for structured securities are based on the principle that their credit ratings should be comparable to bond ratings. The rating agencies use quantitative models to estimate default probabilities or expected losses of CDO tranches. The values of these risk measures are then mapped into letter-grade ratings according to pre-specified bounds corresponding to different rating

[5] This quote and a broader discussion about the meaning of credit ratings is given by Ashcraft and Schuermann (2008).

categories. The S&P's methodology aims to ensure that CDO tranches have the same cumulative (real-world) default probabilities as equally-rated bonds, while the Moody's methodology aims to match (real-world) expected losses. For example, a tranche qualifies for the triple-A rating by Standard and Poor's if its 10-year default probability is equal or less than 0.36%, which is the historical default probability of triple-A corporate bonds over the same time horizon. Similarly, Moody's assigns the triple-A rating to a tranche if its 10-year expected loss is equal or less than 0.0055%, which is the historical loss on equally-rated bonds.[6]

3. Model and assumptions

3.1. Modeling approach

In this part we introduce the market standard method for modeling CDOs. We start with discussing how a CDO structure allocates losses incurred on the underlying assets to the tranches. We then explain the approach to modeling defaults of the collateral bonds. We also define credit ratings and fair spreads of tranches. Last, we discuss how we conduct the study by Monte Carlo simulations.

We construct a CDO backed by a collateral pool consisting of $i = 1, ..., n$ bonds with each bond i having a notional N_i. The total notional of the portfolio is thus equal to $N_{total} = \sum_{i=1}^{n} N_i$. The CDO's maturity time is T. Default times of the obligors are denoted by $\tau_1, \tau_2, ..., \tau_n$ and the corresponding recovery rates are denoted by R_i. The cumulative loss on the collateral pool up to time t is given by:

$$L(t) = \sum_{i=1}^{n} N_i(1 - R_i)\mathbf{1}_{\tau_i<t}, \qquad (1)$$

where $\mathbf{1}$ is the indicator function defined as usual.

The CDO structure splits the total portfolio risk into several tranches with each tranche being defined by its attachment point K_L and its detachment point K_U. Tranche investors cover the portfolio losses exceeding K_L, but limited to the tranche notional $K_U - K_L$. The lower attachment point is also referred to as the tranche subordination level. For example, if the total portfolio notional is \$100 million and the tranche attachment and detachment points are, respectively, \$3 and \$7 million, then the cumulative portfolio losses between \$3 and \$7 million

[6]For details on the S&P rating methodology, we refer to S&P (2002) and S&P (2005). For details on the Moody's methodology, see Moody's (2005) and Moody's (2007).

are passed on as losses to the tranche investors. Formally, the CDO tranche losses up to time t are given by:

$$L(K_L, K_U, t) = \min\left[\max\left(L(t) - K_L, 0\right), K_U - K_L\right]. \tag{2}$$

The key challenge in rating and pricing structured securities lies in modeling defaults of the collateral assets. The appropriate modeling framework must capture not only the univariate risk properties of the underlying assets, but also dependence between defaults of these assets. That is because tranche payoffs are linked to the portfolio loss rate.

The univariate risk properties of the underlying assets are summarized by the cumulative distribution functions of their default times τ_i:

$$F_i(t) = Pr(\tau_i < t) = 1 - S_i(t), \tag{3}$$

where $S_i(t)$ is the survival function to time t. The specification of $F_i(t)$ or $S_i(t)$ depends on the purpose of modeling as it can reflect probabilities either under the physical measure or under the risk-neutral measure. The survival functions together with the recovery rates give all security-specific information needed for analyzing expected cash flows on single-name securities.

Default dependence is modeled using copulas. Let us introduce a series of random variables:

$$V_i = \Phi^{-1}\left(F_i(\tau_i)\right), \tag{4}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. To account for default dependence, we further assume that $V_1, ..., V_n$ jointly follow the multivariate standard normal distribution with specified pairwise correlations between any V_i and V_j. This corresponds to the Gaussian copula approach. For modeling CDOs, the market typically assumes a one-factor model such that the correlations between all collateral assets are due to their exposure to a single common factor. It this case, the V_i can be expressed as:

$$V_i = \sqrt{\rho_i}Y + \sqrt{1-\rho_i}X_i, \tag{5}$$

where $Y \sim N(0,1)$ is the common (systemic) factor, $X_i \sim N(0,1)$ is the idiosyncratic (obligor-specific) component and $\rho_i \in [0,1]$ is the parameter controlling the correlations. It is typical to interpret $V_i \sim N(0,1)$ as the scaled asset value of obligor i, which is in line with the Merton approach of default modeling. The market standard is to assume that all ρ_i are equal to a common ρ, which simplifies the correlation structure. We can then interpret ρ in Eq. 5 as the asset value correlation between any two obligors in the collateral portfolio.

For modeling CDO-squareds, we assume a slightly more complicated correlation structure, which is captured by the two-factor model. We consider a CDO-squared with a collateral pool composed of $j = 1, ..., K$ underlying CDO tranches. In turn, each of the underlying CDO tranches is backed by a portfolio of bonds indexed $i = 1, ..., n$. The scaled value of obligor i belonging to the reference portfolio of the (underlying) jth CDO is denoted as $V_{i,j}$ and can be expressed as:

$$V_{i,j} = \sqrt{\alpha \rho_i} Y + \sqrt{(1 - \alpha) \rho_i} Z_j + \sqrt{1 - \rho_i} X_{i,j}, \tag{6}$$

where Y and ρ_i are as defined below Eq. 5, $X_{i,j} \sim N(0, 1)$ is the idiosyncratic (obligor-specific) component, $Z_j \sim N(0, 1)$ is the factor specific to the reference portfolio of the jth CDO, and finally, parameter $\alpha \in [0, 1]$ determines the relative exposure to the common factor Y and to the CDO-specific factor Z_j. In this setting, the credit risk of underlying tranches is partly driven by tranche-specific factors, which provides additional diversification. If all ρ_i are equal to a common ρ, then ρ gives the asset value correlation between any two obligors within the same CDO collateral pool, while $\alpha\rho$ is the asset value correlation between any two obligors belonging to collateral pools of different underlying CDOs.

3.1.1. Rating Measures

Standard and Poor's ratings are based on tranche default probability, while Moody's ratings are based on expected tranche loss. Tranche default probability is the likelihood that the cumulative portfolio loss exceeds the subordination level of the tranche until maturity time T:

$$PD_{tranche} = \mathbb{E}^P \left(\mathbf{1}_{L(T) > K_L} \right), \tag{7}$$

where P is the physical default probability measure. Expected tranche loss is defined as the loss on the tranche notional until maturity:

$$EL_{tranche} = \frac{\mathbb{E}^P L(K_L, K_U, T)}{K_U - K_L}. \tag{8}$$

We emphasize that credit ratings are determined under the physical measure. The physical measure captures the actual (real-world) default probabilities and it is typically estimated from historical data on default frequencies. The physical measure is the appropriate choice for modeling credit ratings because they are meant to reflect real-world payoff prospects of a security (i.e. real-world default probability or expected loss).

3.1.2. Fair Premia

Holders of a tranche incur losses if the portfolio loss rate exceeds the subordination level of the tranche. The series of cash flows equal to the tranche losses associated with credit events is called the default leg. The present value of the default leg is calculated as:

$$V_{default} = \mathbb{E}^{Q} \int_{0}^{T} B(0,t)\, dL(K_L, K_U, t), \tag{9}$$

where Q is the risk-neutral measure and $B(0,t)$ is the discount factor for the time interval $(0,t)$.

In return for taking on default risk, tranche investors receive premium payments based on the running spread s. The present value of the premium leg is given by:

$$V_{premium}(s) = \mathbb{E}^{Q}\left[\sum_{i=1}^{qT} B\left(0, \frac{i}{q}\right) \frac{s}{q} \left((K_U - K_L) - L\left(K_L, K_U, \frac{i}{q}\right)\right)\right], \tag{10}$$

where q is the frequency of coupon payments (e.g. $q = 4$ for quarterly payments).

Determining the fair spread is equivalent to finding the level of tranche spread, s^*, that equates the default leg and the premium leg. Since the premium leg, as given by Eq. 10, is linear as a function of s, the fair tranche spread equals:

$$s^* = \frac{V_{default}}{V_{premium}(s = 1bp)}. \tag{11}$$

At CDO origination tranche spreads are typically set equal to the fair spread levels such that both sides of the contract have zero value.

For the purpose of calculating fair spreads, we use the risk-neutral measure implied by market information. Risk-neutral default probabilities are typically much higher than their physical counterparts because they incorporate risk premia. The risk-neutral measure can be derived from the term structure of CDS spreads of the collateral bonds given the recovery rate assumptions. The recovery rate estimates are assumed to be exogenous and can be based on the rating agencies' studies of historical data.

3.1.3. Implementation

The aforementioned tranche statistics are most easily calculated using Monte Carlo simulations. In each simulation run, we draw realizations of the random variables Y and X_i from independent standard normal distributions. Next, we compute default times τ_i of the underlying assets by using formulas (5) or (6) and the inverse of formula (4):

$$\tau_i = F_i^{-1}\left(\Phi\left(v_i\right)\right). \tag{12}$$

Once the default times and the corresponding recoveries are determined for all simulation runs, the calculation of tranche default probabilities, expected losses and fair spreads using formulas (7), (8) and (11) is straightforward.

In addition to the fair tranche spreads calculated under the risk-neutral measure Q, we also calculate tranche spreads under the physical measure P. This gives the spreads compensating for default risk in the real-world (pure default risk). Similarly, we calculate tranche default probabilities and expected losses under the risk-neutral measure Q instead of the physical measure P.

3.2. Manufacturing structured assets

Manufacturing structured assets can be decomposed into two steps. The first step is to select the collateral portfolio. The second step is the structuring process. We first discuss how we produce a stylized CDO and then we turn to the CDO-squared case.

3.2.1. CDO collateral portfolio

We choose a homogeneous collateral portfolio of one hundred bonds with a maturity of 10 years. Each bond has a default probability of 10% until maturity, which results in a BBB- rating by Standard and Poor's.[7] We make a simplifying assumption that the survival functions of the underlying bonds have an exponential form, $S_i(t) = e^{-t\lambda_i}$, with a constant default intensity parameter λ_i.[8] The intensity parameter is calibrated by equating the assumed default probability until maturity (e.g. $p_i = 10\%$) to the default probability implied by the exponential survival function: $p_i = 1 - e^{-T\lambda_i}$.[9]

We assume that the collateral bonds have random recovery rates drawn from a Beta distribution with a mean of 50% and a standard deviation of 20%, which implies that the bond's expected loss is 5%. Hence, according to the Moody's

[7]A BBB- bond rated by S&P has a 10-year default probability between 5.88% and 10.64% according to the S&P benchmark tables (S&P, 2005).

[8]The exponential (real-world) survival functions are a good approximation when compared to exact survival functions provided by S&P (2005).

[9]The intensity parameter for the risk-neutral default probability is calibrated similarly.

criteria the bonds qualify for the Ba1 rating.[10]

The ability to produce highly rated tranches is critically dependent on the joint default behavior of their collateral assets. According to the Standard and Poor's rating assumptions, the asset value correlation between corporate obligors belonging to different industry sectors is 5%, while the asset value correlation within an industry sector is 15% (for U.S. bonds), see S&P (2005). We set the asset value correlation to 12.5%, which is a realistic average correlation if the bonds belong to several industries.[11]

We also assume that the market-implied default probability of each of the collateral bonds is equal to 20% until maturity of 10 years, which is double the physical probability. Such assumption is equivalent to a market spread of 111.95 bps on the collateral bonds.[12] Moreover, it is also in line with the studies on the relationship between physical and risk-neutral default probabilities in the corporate bond markets. The literature suggests that risk-neutral default probabilities are 2 to 5 times higher than their physical counterparts for BBB rated bonds, see Berndt et al. (2005), Driessen (2005), Delianedis and Geske (2003) or Hull et al. (2005). Therefore, our choice of the multiplier is equal to the lower bound of estimates given in the literature. In the context of this study, it is a very cautious assumption because if we had chosen a higher multiplier, the results of this paper would be stronger (i.e. the tranche yield enhancement would be higher).

3.2.2. CDO structuring

The capital structure of the CDO is chosen for the purpose of minimizing the cost of financing of the underlying debt. Given investors' reliance on credit ratings, the cost of financing of a tranche is decreasing in its credit quality. This leads to clear incentives to maximize the volume of tranches with as good ratings as possible. The market practice is to look at the criteria of the rating agencies and to produce tranches that just qualify for their credit ratings. The structuring of CDO tranches is therefore strongly interrelated with the rating process.

We first describe the junior mezzanine tranche, which is tailored to have iden-

[10] A Ba1 bond rated by Moody's has a 10-year expected loss between 3.25% and 5.17% (Moody's, 2007).

[11] In practice, the average correlation between collateral assets is typically lower than 12.5% because collateral portfolios include not only U.S. bonds, but also European or Asian bonds as well as RMBS or ABS tranches. For asset value correlation lower than 12.5%, the yield enhancement on tranches is even higher as demonstrated in the sensitivity analysis in Section 5.

[12] We also assume a fixed discount factor of 2% per annum.

tical credit quality as the underlying corporate bonds. It means that we not only ensure that this tranche has the same credit ratings as the underlying bonds, but we also impose a stronger condition that this tranche and the bonds have the same (real-world) default probabilities and (real-world) expected losses. This is very convenient for our further analysis of the rating-premia relationship, but it slightly departs from the typical structuring process. The lower attachment point of the junior mezzanine tranche is chosen as a 90% quantile of the real-world portfolio loss distribution such that the 10-year tranche default probability is 10%. Next, we fix the upper attachment point such that the 10-year expected tranche loss is 5%. For our portfolio, this implies that the lower and upper attachments points of the tranche are 9.90% and 14.75%, respectively. It follows from the obtained tranche default probability and expected loss that the tranche receives a BBB- rating from S&P and Ba1 from Moody's.

The more senior tranches are tailored in line with the market practice of maximizing the size of tranches with the highest ratings. While we report ratings of both S&P and Moody's, the structuring for these more senior tranches is based solely on the S&P criteria. For the S&P methodology, which measures tranche default probability, only the lower tranche attachment points matter for determining credit ratings. Therefore, we first choose the subordination level of the super-senior AAA tranche as a quantile of the real-world portfolio loss distribution such that this tranche meets the benchmark 10-year default probability of 0.36%.[13] Similarly, we construct the senior AA tranche by choosing its lower attachment point such that the tranche meets the 10-year default probability target of 0.87%. The upper attachment point of the AA tranche is the lower attachment of the super-senior tranche. For our portfolio, these two tranches have subordinations of 17.08% and 19.45%, respectively.

We also obtain two other tranches, which have both attachment and detachment points implied by the tranches defined so far. The first one is the unrated equity tranche, which is at the bottom of the capital structure. Its lower attachment is 0% and its upper attachment is given by the subordination level of the junior mezzanine tranche, i.e. 9.90%. Another tranche is in between the junior mezzanine tranche and the senior tranche, so it goes from 14.75% to 17.08% of the CDO notional. It is rated A- by S&P and Baa1 by Moody's.

[13] Starting from 2005, S&P uses different default probability benchmarks for CDO tranches, which are no longer based on historical bond performance. After the change, senior CDO tranches have higher target default probabilities corresponding to historical tranche performance; however, we still use the corporate bond benchmarks to preserve direct comparability of ratings.

Table 1: CDO tranche risk statistics, ratings and premia.

This table reports the results of CDO structuring, rating and tranche pricing. The first two columns summarize the capital structure of the CDO. Next, the table reports tranche default probabilities, expected losses (over a 10-year horizon) and annualized spreads calculated under the physical measure and the risk-neutral measure. The physical measure corresponds to the assumption of 10% default probability of the underlying bonds (over a 10-year horizon), whereas the risk-neutral measure corresponds to default probability of 20%. The 'Physical measure' part of the table is related to the rating process, so in columns (3) and (4) we also report credit ratings by S&P and Moody's. Column (5) reports the spreads compensating for pure default risk. From the risk-neutral results, the most important is column (8), which gives the fair (market) spread. The last row of the table shows the statistics for the underlying corporate bonds.

		Physical measure (PD=10%)			Risk-neutral measure (PD=20%)		
Tranche	Tranche subordi-nation	Default probability & S&P rating	Expected loss & Moody's rating	Spread (bps)	Default probability	Expected loss	Fair spread (bps)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tranche 1 equity	0.00%	98.33% 'NR'	47.50% 'NR'	636.54	99.90%	78.53%	1475.40
tranche 2 junior mezz.	9.90%	10.00% 'BBB-'	5.00% 'Ba1'	48.25	44.64%	30.24%	320.69
tranche 3 senior mezz.	14.75%	1.97% 'A-'	1.35% 'Baa1'	12.76	18.43%	14.55%	143.83
tranche 4 senior	17.08%	0.87% 'AA'	0.58% 'A2'	5.43	11.09%	8.46%	81.81
tranche 5 super-senior	19.45%	0.36% 'AAA'	0.01% 'Aa1'	0.10	6.21%	0.27%	2.52
corporate bond	n.a	10.00% 'BBB-'	5.00% 'Ba1'	53.06	20.00%	10.00%	111.95

Table 1 presents the results of structuring and rating in columns (1)-(5). These results are obtained under the physical measure, which is appropriate for modeling credit ratings. We can see how CDO prioritization of cash flows adjusts the risks of tranches. The default probabilities and expected losses of the tranches are decreasing with tranche seniority. Most of the credit risk is contained in the equity tranche, which absorbs all portfolio losses up to the limit of 9.90% of the CDO notional. The super-senior AAA tranche has roughly 80% of the deal notional. The remaining columns of Table 1 present the risk-neutral results, which are relevant for tranche pricing as discussed in the next section.

A similar CDO structuring exercise was done by Krahnen and Wilde (2008) who use (Moody's) historical default rates to determine tranche subordination levels as quantiles of the portfolio loss distribution. The practice of choosing tranche subordinations to just meet the rating criteria is common in the market for CDOs backed by non-synthetic assets. In contrast, tranche subordination levels are standardized and pre-defined in the markets for CDS index tranches. For example, the iTraxx Europe index is divided into six tranches with consecutive subordinations of 0%, 3%, 6%, 9%, 12%, and 22%.

3.2.3. Creating a CDO-squared

We also analyze CDO-squared securities, which have incurred particularly large losses during the financial crisis. CDO-squareds are created by re-securitizing CDO tranches for which there is limited market demand. These are typically tranches rated A+ or lower.

CDO-squared tranches are rated according to the same principles as CDOs. The market standard is to use the 'the bottom up' approach, which derives the cash-flows on the underlying tranches directly from the performance of their collateral bonds. This approach accounts for specific characteristics (e.g. credit quality) and overlap among the collaterals of the underlying CDO tranches.

We choose a CDO-squared collateral pool composed of thirty mezzanine BBB- tranches. Each BBB- tranche comes from the stylized CDO deal described in the previous subsection. We assume that the underlying BBB- tranches reference portfolios of different bonds, so there is no overlap among their collateral portfolios. We further assume that the asset value correlation between any two obligors within the same CDO collateral pool is 12.5%, while the asset value correlation between any two obligors belonging to collaterals of different underlying CDOs is 3.5%. This is equivalent to assuming that in Eq. 6 parameter $\rho = 12.5\%$ and $\alpha\rho = 3.5\%$. We thus ensure additional diversification at the level of the underlying tranches, which is critical for the ability to produce highly rated CDO-squared tranches. A similar approach was used by Hull and White (2010) in the analysis of ABS CDOs. In practice, such diversification can be achieved by selecting tranches backed by collateral pools that have different industry concentrations and geographic location.[14] In addition, collateral pools of CDO-squareds very often include some tranches of asset backed securities (e.g. RMBS or ABS).

We choose the capital structure of the CDO-squared to ensure that its tranches have similar credit quality to the corresponding tranches of the stylized CDO. For

[14]For example, S&P assumes a correlation of 0% between two corporate bonds belonging to different industry sectors and different regions (regions are defined as Asia, Europe etc.), see S&P (2005).

this purpose, we apply the same structuring scheme as in the CDO case, but to the CDO-squared collateral pool. The results of structuring for the CDO-squared are reported in columns (1) – (5) of Table 2. Due to the assumed structuring process, the corresponding CDO and CDO-squared tranches have the same credit ratings from both S&P and Moody's. In particular, the junior mezzanine tranche of the CDO-squared is tailored to have a default probability of 10% and an expected loss of 5%, which results in a BBB- rating by S&P and a Ba1 rating by Moody's. We also produce two CDO-squared tranches with AAA and AA ratings by S&P, which just meet the default probability benchmarks of 0.87% and 0.36% required for these rating categories. Finally, we obtain the senior mezzanine tranche and the equity tranche, which have the attachment and detachment points implied by other tranches of the CDO-squared.

4. Credit ratings and fair premia

In this section we analyze the relation between credit ratings and fair premia. We start with calculating tranche default probabilities, expected losses and spreads under the assumption that the market-implied default probability of each underlying bond is 20% until maturity, which implies a market spread of 111.95 bps. The obtained results are reported in columns (6)-(8) of Tables 1 and 2 for the CDO and CDO-squared, respectively.

A number of observations can be made. Firstly, it is seen that the transition from the physical to the risk-neutral measure corresponds to a huge increase in the default probabilities, expected losses and spreads for all tranches. Secondly, the magnitude of the changes, regardless of the tranches considered, is much higher for the CDO-squared than for the CDO. Thirdly, from a market pricing perspective, the most important observations follow from the analysis of the junior mezzanine tranches. That is because these tranches and the collateral bonds have the same (real-world) default probabilities and (real-world) expected losses, which allows us to separate the rating-premia relation from any differences in credit quality. In Table 1 we see that while the underlying portfolio of BBB- bonds has a fair spread of 111.95 bps, the similarly-rated CDO tranche has a fair spread of 320.69 bps. Hence the fair spread is almost 3 times as high. In Table 2 we observe that the corresponding CDO-squared tranche rated BBB- has a fair spread of 749.52 bps, which is almost 7 times higher than the spread on the similarly-rated underlying bonds. Thus it is clear that fair spreads on tranches are much higher than fair spreads on corporate bonds even when there are absolutely no differences in

Table 2: CDO-squared tranche risk statistics, ratings and premia.

This table reports the results of CDO-squared structuring, rating and tranche pricing. The first two columns summarize the capital structure of the CDO-squared. Next, the table reports tranche default probabilities, expected losses (over a 10-year horizon) and annualized spreads calculated under the physical measure and the risk-neutral measure. The physical measure corresponds to the assumption of 10% default probability of the underlying bonds (over a 10-year horizon), whereas the risk-neutral measure corresponds to default probability of 20%. The 'Physical measure' part of the table is related to the rating process, so in columns (3) and (4) we also report credit ratings by S&P and Moody's. Column (5) reports the spreads compensating for pure default risk. From the risk-neutral results, the most important is column (8), which gives the fair (market) spread. The last row of the table shows the statistics for the underlying corporate bonds.

		Physical measure (PD=10%)			Risk-neutral measure (PD=20%)		
Tranche	Tranche subordination	Default probability & S&P rating	Expected loss & Moody's rating	Spread (bps)	Default probability	Expected loss	Fair spread (bps)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tranche 1 equity	0.00%	77.88% 'CCC-'	32.46% 'Caa2'	338.84	99.62%	91.87%	1498.01
tranche 2 junior mezz.	13.27%	10.00% 'BBB-'	5.00% 'Ba1'	46.89	80.60%	68.16%	795.71
tranche 3 senior mezz.	24.92%	2.07% 'A-'	1.38% 'Baa1'	12.86	55.67%	49.27%	520.66
tranche 4 senior	31.25%	0.87% 'AA'	0.58% 'A2'	5.36	43.09%	37.50%	379.86
tranche 5 super-senior	37.50%	0.36% 'AAA'	0.04% 'Aa1'	0.40	32.21%	7.61%	71.83
corporate bond	n.a	10.00% 'BBB-'	5.00% 'Ba1'	53.06	20.00%	10.00%	111.95

credit quality.

A closer examination of the results reveals that the yield enhancement is attributable to concentration of risk premia in spreads of non-equity tranches. Fair spreads on credit-sensitive instruments consist of compensation for default risk in the real-world (pure default risk) and additional risk premia. Only pure default risk is closely related to credit ratings, while risk premia compensate investors for the uncertainty about securities' payoffs. The compensation for pure default risk can be read in column (5) of Table 1 or Table 2. The risk premia are simply calculated by subtracting the compensation for pure default risk from the total fair spread reported in column (8). For the corporate bonds, the assumed spread of 111.95 bps can be decomposed into 53.06 bps of pure default risk compensation and 58.89 bps of risk premia. For the junior mezzanine tranches, the compensations for pure default risk are 48.25 bps and 46.89 bps, respectively, for the CDO and CDO-squared. These values are slightly lower compared to 53.06 bps of pure default risk compensation for the corporate bonds. In contrast, the risk premia on these tranches are much higher and equal to 272.44 bps and 748.82 bps, respectively. Relative to the similarly-rated corporate bonds, the risk premia are thus multiplied by a factor of almost 5 for the CDO tranche and by a factor of 13 for the CDO-squared tranche.

The results for the junior-mezzanine tranches are striking. They demonstrate that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds, which means that credit ratings are by far insufficient for pricing. The yield enhancement is possible because risk premia are concentrated in non-equity tranches, while the rating methodologies capture solely pure default risk. In other words, structured finance produces securities that have low pure default risk and thus obtain high credit ratings, but have inherently high risk premia. On the one hand, it allows investors to earn higher spreads on highly rated portfolios. On the other hand, investors who overly rely on credit ratings for inferring fair spreads are likely to accept insufficient risk compensation on structured products.

The foregoing analysis can be generalized to the case of the more senior tranches; however, it requires consideration of similarly-rated corporate bonds. For this purpose, we create bonds that have identical (real-world) default probabilities and (real-world) expected losses as the corresponding tranches, which implies the same credit quality. We call these bonds *risk-equivalent* to the respective tranches.[15] To determine fair spreads on the risk-equivalent bonds, we assume that their risk-neutral default probabilities are double the historical probabilities regardless of credit quality (robustness to this assumption is explained further down). The obtained results are summarized in Table 3. In column (6) we report fair spreads on the risk-equivalent bonds, while in columns (1)-(5) we summarize tranche ratings and fair premia previously shown in Tables 1 and 2.

The main message from Table 3 is that fair premia on the non-equity tranches are much higher than fair premia on their risk-equivalent bonds. The magnitude of the yield enhancement critically depends on whether the tranche belongs to the CDO or CDO-squared and it also varies with tranche seniority. For example, the

[15]Since the expected losses and default probabilities of the corresponding CDO and CDO-squared tranches are different, we construct risk-equivalent bonds separately for the CDO and CDO-squared. In most cases the differences are small.

Table 3: Comparison of fair spreads on tranches and risk-equivalent bonds.

In columns (1)-(5) we summarize tranche ratings and fair spreads reported previously in Tables 1 and 2. Note that the corresponding tranches and their risk-equivalent bonds have the same credit ratings by S&P and Moody's. In column (6) we report fair spreads on risk-equivalent corporate bonds (separately for CDO / CDO-squared). The risk-equivalent bonds are defined as having the same (real-world) default probabilities and (real-world) expected losses as the corresponding tranches. To calculate fair spreads on these bonds, we assume that their risk-neutral default probabilities are double the physical probabilities.

Tranche	S&P rating	Moody's rating	Fair spread (bps)		
			CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	'BBB-'	'Ba1'	320.69	795.71	111.95 / 111.95
tranche 3 senior mezz.	'A-'	'Baa1'	143.83	520.66	28.03 / 28.23
tranche 4 senior	'AA'	'A2'	81.81	379.86	11.62 / 11.83
tranche 5 super-senior	'AAA'	'Aa1'	2.52	71.83	0.22 / 0.83

spread on the super-senior CDO tranche is equal to 2.52 bps, while the spread on the corresponding risk-equivalent AAA bond is equal to 0.22 bps. For the CDO-squared, the spread on the super-senior tranche is as high as 71.83 bps, while the corresponding risk-equivalent AAA bond yields 0.83 bps.

An argument can be made that the spreads on the risk-equivalent bonds reported in Table 3 are underestimated. These bond spreads are calculated under the assumption that risk-neutral default probabilities of all bonds are double their physical probabilities; however, our results are fairly robust to this assumption. The market evidence suggests that for highly rated bonds the ratio of risk-neutral to physical default probabilities can be much higher. For example, Hull et al. (2005) report a ratio of 16.8 for AAA bonds. If we had assumed such ratio in Table 3, then the AAA bond, which is risk-equivalent to the super-senior CDO tranche, would have a spread of 1.88 bps instead of 0.22 bps. That is still lower than the fair tranche spread of 2.52 bps. In case of the super-senior CDO-squared tranche, the same ratio would imply a fair spread of 7.01 bps on the risk-equivalent AAA bond, which is much lower than the tranche spread of 71.83 bps.

In the appendix in Table A.I we replicate the results of Table 3 under the assumption of 180.72 bps market spread on the collateral BBB- bonds, i.e. a higher bond risk premium as suggested by Hull et al. (2005).[16] These results indicate that with increasing risk premia on the underlying bonds, relative to the baseline assumption, the yield enhancement on tranches is increasing as well. For example, it is seen in Table A.I that the fair spreads on the super-senior CDO and CDO-squared tranches are equal to 13.46 bps and 428.03 bps, respectively, compared to 2.52 bps and 71.83 bps in the baseline case.[17]

The documented large differences in fair spreads between similarly-rated tranches and bonds create opportunities for rating arbitrage, which means that excess spreads can be distributed between tranche investors and CDO issuers. This explains why structured finance securities can be so appealing to both originators and investors. Since investors are typically risk averse and CDO tranches are tailored to their risk appetites, the total risk compensation paid on the tranches of a CDO can be lower than the total spread received on the collateral portfolio. In other words, the risk-return profiles of the tranches can be attractive to investors at the spread levels, which are below the model-implied fair spreads. In this sense the ability of financial engineering to tailor the risks of tranches creates value. The remaining share of the yield can then be allocated to CDO issuers compensating them for the risks associated with their part of structured finance activities. These risks arise because very often the originators are unable to sell the total notional of all CDO tranches and have to retain and hedge the remaining risks. For example, in the market for synthetic CDOs, single tranche issues were very popular, so the originating banks were only partly securitizing the underlying CDS portfolios. There are also reputational risks, which can lead financial institutions to even bail-out their CDOs as was in the case of Bear Stearns.

The analysis done in this paper assumes that credit ratings and fair spreads are accurate and unbiased for all securities under consideration. This follows from our theoretical approach where tranche risk measures and spreads are calculated using the market-standard models on the basis of (realistic) assumptions. We are not concerned about a possible divergence between the true-world and the models because the analysis is limited to the stylized setting. Since we use the same

[16]The market spread of 180.72 bps is equivalent to assuming that risk-neutral default probability of each of the underlying BBB- bonds is triple the 10-year physical default probability of 10%. That is motivated by Hull et al. (2005) who find that the ratio of risk-neutral to physical default probabilities is equal to 5.1 for BBB bonds and 2.1 for BB bonds.

[17]We do not replicate the results of Table 3 under the assumption that market spread on the collateral bonds is lower than 111.95 bps. That is because this baseline level of spread is rather low for BBB- bonds as discussed in Section 3.

assumptions for both rating and pricing securities, our results are consistent and they illustrate fundamental limitations of the rating methodologies. We show that even if the rating agencies correctly estimate the real-world default probabilities and/or expected losses of CDO tranches, these two risk measures are not very informative about the level of fair spreads.

Several recent studies attribute the failings of credit ratings to mistakes made by the rating agencies, for example, pointing out to overly optimistic rating assumptions or failure to account for parameter uncertainty (e.g. Coval et al., 2009b). Undoubtedly, during the financial crisis, credit ratings of structured instruments have failed badly as an indication of payoff prospects. Structured products experienced many more downgrades than corporate bonds and their downgrades were also very severe, particularly for triple-A tranches (see Benmelech and Dlugosz (2009b) for a discussion on performance of CDO ratings). We point out that the problems with the current rating methodologies go beyond difficulties in implementation of otherwise correct methodologies. The key challenge comes from the fact that the rating methodologies do not capture risk premia, which are (typically) much higher for CDO tranches than for similarly-rated corporate bonds. While the standard rating approach has proven to be adequate for corporate bonds, structured securities might require a different rating approach because they are specifically tailored by originators to maximize the yield enhancement on tranches.

Throughout the paper we interpret the spreads computed under the risk-neutral measure as the market fair spreads. These spreads are calculated using the Gaussian copula model with the same correlation parameter for all tranches. The market practice for pricing is to use different correlations for different tranches, which are summarized by the base correlation curve.[18] The use of this curve can be interpreted as a way to correct for limitations of the Gaussian copula default dependence structure, market appetite for risk or tranche exposure to systemic risk. Thus our approach departs from the actual market pricing, but it captures the economics of CDO yield enhancement and rating arbitrage.

5. Sensitivity analysis

We examine the sensitivity of tranche payoffs to default probabilities of the underlying bonds with the aim of providing a clear-cut explanation of the yield

[18]The base correlation curve is a standard way of quoting prices in the markets for CDS index tranches, which are traded by professional parties.

enhancement on tranches as well as illustrating risk properties of tranches. We focus on analyzing sensitivity of tranche default probabilities and expected tranche losses because these risk measures determine credit ratings, while expected losses are also closely related to spreads.[19] Thus by analyzing expected losses we can also infer about spreads. Fair spreads are implied by the risk-neutral level of expected losses, while spreads compensating for default risk in the real-world are determined by expected losses under the physical measure.

Figure 1 presents the sensitivity results for CDO tranches. To benchmark tranche sensitivities, we plot curves corresponding to the underlying portfolio of bonds. To facilitate inference, we add vertical lines representing the real-world measure, i.e. at the collateral default probability equal 20%, and the risk-neutral measure, i.e. at the collateral default probability equal 10%. These lines cross the CDO tranche curves at the values corresponding to the results of Table 1.

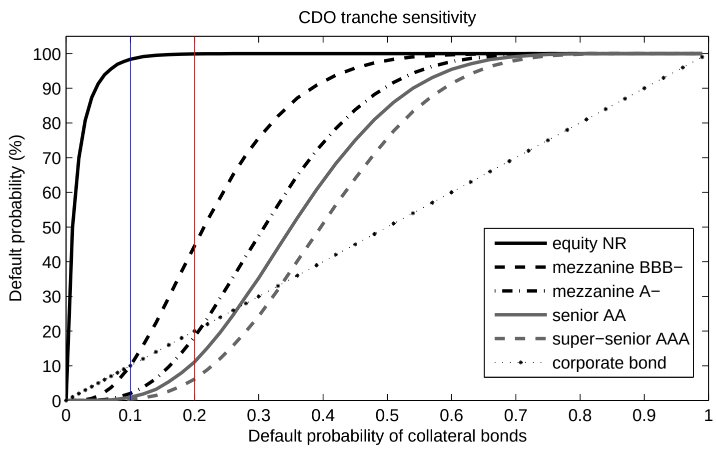
Panel A of Figure 1 explores the sensitivity of tranche default probabilities to changes in default probabilities of the collateral bonds. It is seen that the sensitivity of the tranche default probabilities is generally higher than the corresponding sensitivity of the collateral bonds. In Panel B we present the sensitivity of expected tranche losses and we observe qualitatively similar results. Only the expected loss of the super-senior tranche appears to be fairly insensitive to a modest increase in the collateral default probability. However, the relative (percentage) increase in the expected loss of the super-senior tranche is very large in the 10-20% interval of the collateral default probability. Table 1 shows that the expected tranche loss increases 27 times (from 0.01% to 0.27%) when the collateral default probability doubles (from 10% to 20%).

Figure 1 indicates that the key to understanding the mechanics of the yield enhancement on tranches lies in high sensitivity of expected tranche losses to default probability of the underlying bonds. To illustrate the argument, let us consider the junior mezzanine tranche and the (underlying) corporate bond. Clearly, these two securities have equal expected losses at the real-world level of the collateral default probability ('0.1' line). In contrast, the expected tranche loss at the risk-neutral level of the collateral default probability ('0.2' line) is considerably higher than the expected bond loss. That is because the curve of the expected tranche loss is steeper than the curve of the expected bond loss in the 10-20%

[19]It follows from Eq. 8 and Eq. 9 that the expected tranche loss is equal to the tranche default leg rescaled by the tranche notional and corrected for discounting. This means that there is close to a one-to-one relation between expected tranche losses and tranche spreads.

interval of the collateral default probability. Consequently, the fair spread on the junior-mezzanine tranche is much higher than the fair spread on the corporate bonds. Similar reasoning applies to the case of the more senior tranches. How-

Panel A: Sensitivity of CDO tranche default probabilities



Panel B: Sensitivity of CDO expected tranche losses

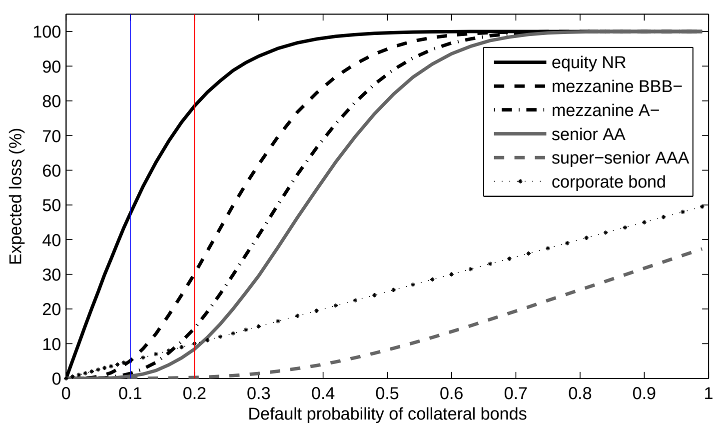


Figure 1: This figure illustrates sensitivity of payoff prospects of CDO tranches to default probabilities of the collateral bonds. Panel A presents sensitivity of tranche default probabilities, whereas Panel B presents sensitivity of expected tranche losses. The vertical lines at value '0.1' correspond to the real-world level of collateral default probabilities, whereas the '0.2' lines correspond to the risk-neutral level. For ease of comparison we add curves representing the collateral bonds.

ever, these tranches cannot be directly compared to the collateral bonds as they are of different credit quality. Nevertheless, the yield enhancement on the more senior tranches is similarly driven by the difference between the real-world and the risk-neutral levels of expected losses. This difference strongly depends on the sensitivity of expected tranche losses analyzed in Figure 1.

The yield enhancement mechanics for CDO-squared tranches is analogous to that for CDO tranches. It is evident from Figure 2 that the sensitivity of CDO-squared tranches is much higher compared to the CDO case considered earlier. It is seen that the non-equity CDO-squared tranches are structured to meet very low default probability and expected loss benchmarks that are required for their rating categories. However, as soon as the default probability of the collateral bonds increases beyond the 10% real-world level, we note a huge increase in both risk measures of the tranches. Particularly, the largest increments in the tranche default probabilities and expected tranche losses are visible in the 10-20% interval of the collateral default probability, which explains the magnitude of the yield enhancement on the CDO-squared tranches.

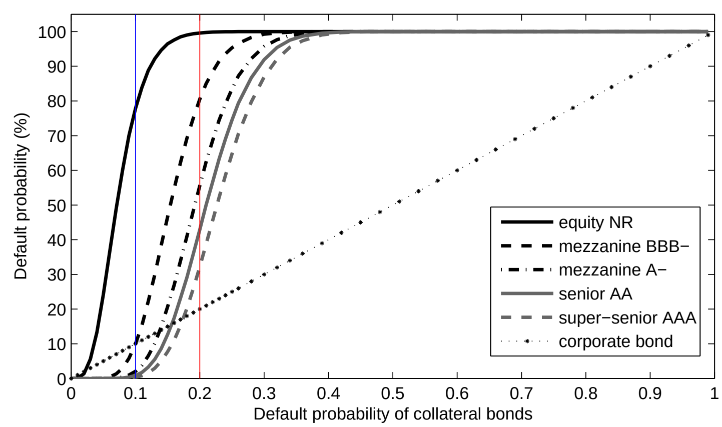
Figures 1 and 2 clearly demonstrate that the higher the sensitivity of tranche payoffs, the higher the yield enhancement on tranches. This sensitivity depends on characteristics of the collateral pool such as portfolio diversification. Portfolio diversification can be increased by choosing assets that are less correlated or by increasing the number of assets while keeping the portfolio notional fixed. We find that if we had chosen a more diversified portfolio, then the tranche sensitivity curves presented in Figures 1 and 2 would be steeper.[20] From Table A.II in the appendix we see that for asset value correlation of 5%, the junior mezzanine tranche has a fair spread of 522.85 bps and the corresponding CDO-squared tranche has a fair spread of 1271.46 bps compared to, respectively, 320.69 bps and 749.52 bps in the baseline case of 12.5% correlation. We thus establish that tranches backed by more diversified portfolios have higher fair spreads than tranches backed by less diversified portfolios, ceteris paribus. These findings sharply contrast with the widespread view that high-diversification of a collateral pool is an advantage for CDO investors. This view is also shared by the rating agencies; for example, when S&P discusses strengths and weaknesses of a newly issued CDO[21], then high diversification of the collateral portfolio is always classified as a 'strength'.

[20] Higher portfolio diversification means that the probability of large portfolio losses is lower, which results in lower tranche subordination levels for all non-equity tranches.

[21] Upon announcing credit ratings of newly issued CDO tranches, S&P releases so-called 'New Issue' reports that provide S&P's analysis and justification for the assigned ratings.

The sensitivity of tranche payoffs can be associated with tranche leverage. Tranches are highly leveraged when their expected payoffs change a lot in response to changes in credit conditions. Figures 1 and 2 clearly demonstrate that

Panel A: Sensitivity of CDO-squared tranche default probabilities



Panel B: Sensitivity of CDO-squared expected tranche losses

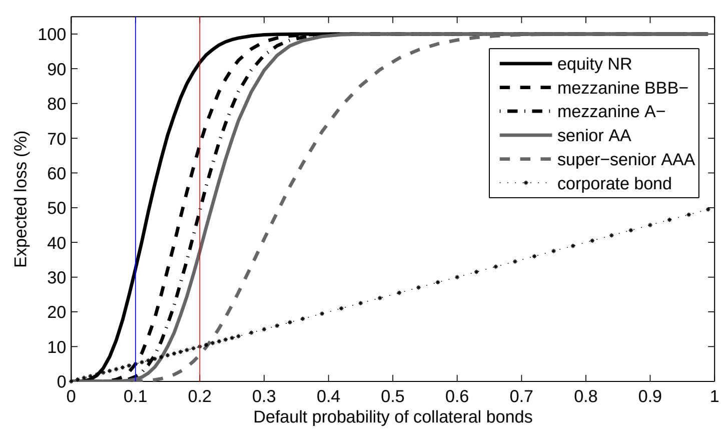


Figure 2: This figure illustrates sensitivity of payoff prospects of CDO-squared tranches to default probabilities of the collateral bonds. Panel A presents sensitivity of tranche default probabilities, whereas Panel B presents sensitivity of expected tranche losses. The vertical lines at value '0.1' correspond to the real-world level of collateral default probabilities, whereas the '0.2' lines correspond to the risk-neutral level. For ease of comparison we add curves representing the collateral bonds.

CDO-squareds are much more leveraged than CDOs. Figure 2 also shows that risk properties of CDO-squared tranches are very different from the risk properties of bonds. Particularly, the expected loss curves of the CDO-squared tranches become very steep once the collateral default probability exceeds the real-world level. It means that CDO-squared tranches have little upside potential relative to their real-world expected payoffs, while adverse market changes are likely to lead to huge losses. We can thus say that CDO-squareds are structured at the 'critical' levels of collateral default probability, which is a result of tailoring the tranches to just meet the rating agencies' criteria. Similar asymmetry of payoffs is seen in Figure 1 for CDO tranches although it is less pronounced. In contrast, a pool of bonds is characterized by symmetry in payoff prospects in the sense that the upside potential is of the same magnitude as the downside potential.

In a more general setup compared to the foregoing analysis, the risk-neutral default probabilities of the underlying bonds are implied by market information (i.e. CDS spreads). For illustration, let us keep the assumption that the underlying portfolio is composed of BBB- rated bonds. We now relax relax the assumption that this BBB- portfolio has a risk-neutral default probability of 20% and we analyze implications of the fact that CDO issuers can select collateral BBB- bonds with different CDS spreads.

The rating agencies derive physical default probabilities on the basis of credit ratings, so by construction these probabilities are equal for similarly-rated bonds. For BBB- rated bonds, the real-world default probability is equal to 10%. However, CDS spreads are characterized by substantial variation even between similarly-rated bonds. That is because the pricing of risk includes market risk premia in addition to pure default risk. It is evident from Figure 1 and Figure 2 that the higher the risk-neutral default probabilities of the collateral bonds, the higher the yield enhancement on tranches.[22] A CDO issuer who aims to maximize rating arbitrage on tranches should therefore select collateral bonds with relatively high CDS spreads for their credit ratings. Such practice leads to the adverse selection problems as high CDS spreads indicate higher risks that are not captured by the rating agencies. There is anecdotal evidence that such adverse selection was one of the reasons for the poor performance of CDOs during the financial crisis (Fitch, 2008).

[22]This observation holds for risk-neutral default probabilities within the realistic range up to roughly 40% default probability over a 10 year perdiod.

6. Rating and price stability of CDO tranches

In this section we analyze the stability of ratings and prices of CDO tranches. Figures 1 and 2 illustrate that even a highly rated tranche, which is structured to have a minute expected loss under the physical measure, can incur heavy losses if the realized default rate of the collateral pool exceeds the assumed rate. In a dynamic setting, tranche prices might become depressed even prior to the realization of collateral losses. When credit conditions deteriorate, then CDS spreads widen as a consequence of a rise in actual default probabilities as well as corresponding risk premia. Investors should then re-price CDO tranches using the revised market-implied default probabilities of the underlying bonds. The changes in prices of CDO tranches are typically much higher than the changes in prices of corporate bonds due to the high sensitivity of tranche payoffs documented in Section 5.

In case of unfavorable market conditions, tranche ratings can be expected to come under severe stress as well. Hereby, we enumerate key factors that are likely to cause a deterioration in tranche ratings. Firstly, credit ratings are highly sensitive to credit enhancement levels, which are reduced once defaults hit the underlying portfolios. Secondly, possible downgrades within a collateral pool lead to an increase of the rating agencies' estimates of (real-world) default probabilities of the collateral bonds. This has a stronger effect on tranche ratings than on bond ratings due to higher sensitivity of tranche payoffs. Thirdly, the re-pricing of CDO tranches, which is typical during unfavorable market conditions, is likely to trigger further downgrades of CDO tranches. That is because the rating agencies will feel the pressure to revise tranche ratings as their market prices fall. Otherwise, the resultant disparity between high credit ratings of CDO tranches and their low market value would (at some point) undermine the reliability of the rating agencies.

To illustrate the divergence between the stability of CDO tranches and corporate bonds, we analyze a scenario corresponding to a fairly severe deterioration in credit conditions, i.e. a one notch downgrade of the entire collateral portfolio from BBB- to BB+. The CDO and CDO-squared are structured and rated under the baseline assumptions discussed in Section 3. We assume that soon after the issuance, the estimates of 10-year default probabilities of the underlying bonds increase from 10% to 13%. The tranches are next re-rated using the revised default probabilities of the collateral bonds, but keeping the tranche subordination levels fixed. Table 4 presents the results for the CDO and CDO-squared.[23]

Table 4 documents a dramatic deterioration in the credit quality of the tranches, particularly for the senior and super-senior tranches.[24] The super-senior CDO tranche is downgraded from the initial rating of AAA to AA- and the corresponding CDO-squared tranche is downgraded as far as to the BBB+ grade. In other words, a one notch downgrade of the collateral pool triggers downgrades of the super-senior tranches by as many as 3 and 7 notches.

An argument can be made that the scenario analyzed in Table 4 is not very realistic as the rating agencies are unlikely to downgrade the entire collateral portfolio and subsequently re-run their rating models using the revised default probabilities. However, we note that a similar deterioration in credit quality of the tranches can occur if a large portion of the collateral bonds is downgraded by more than one notch. Furthermore, if one considers default contagion, then a default of a single bond within the collateral portfolio can by itself explain a substantial increase in default probabilities of the surviving bonds.

To examine the impact of default contagion, we consider a scenario when a single default within the collateral portfolio occurs soon after CDO origination. Let us first assume that this early default does not change the market expectations about default probabilities of the surviving bonds. In this setting, the possible impact on tranche ratings is limited because the portfolio loss rate increases by about 0.5% (given a 50% recovery rate). It might trigger, for example, a one notch downgrade of the tranches if they were tailored to just satisfy the rating criteria.[25]

The impact of a single default is likely to be substantially different in the presence of credit contagion when an early default signals a low realization of the common economic factor Y in Eq. 5 or Eq. 6.[26] The credit-worthiness of the tranches is then reduced not only due to a partial loss of tranche credit enhancements, but also due to the increased likelihood of a market-wide deterioration in

[23]The values reported in Table 4 correspond to those at which tranche curves in Figures 1 and 2 would cross vertical lines at 10% and 13% of the collateral default probability.

[24]In this section, for ease of exposition, we consider only S&P ratings.

[25]Nevertheless, tranches are highly sensitive to several defaults within a collateral pool, which follows from the sensitivity analysis in Section 5.

[26]In this paper we consider information-driven default contagion; for a discussion see Schonbucher (2004). This means that default contagion arises because an obligor's default reveals some information about the common economic factor Y driving the riskiness of all obligors. This definition is different from a more standard one according to which default contagion is associated with "a direct causal relationship between two obligor's defaults" (Schonbucher, 2004).

Table 4: Impact of a deterioration in collateral credit quality on tranche ratings.

The table analyzes the impact of a deterioration in credit conditions that corresponds to an increase in 10-year default probabilities of the collateral bonds from 10% to 13% (i.e. a one-notch downgrade from BBB- to BB+). The first two columns summarize the capital structure of the CDO and CDO-squared. Next, in columns (4) and (5) we report tranche default probabilities under the standard market conditions (i.e. 10% collateral PD). The CDO and CDO-squared are structured and rated under the standard market conditions, so in columns (4) and (5) we also report S&P ratings. In columns (7) and (8) we report tranche default probabilities after the market conditions have deteriorated (i.e. 13% collateral PD). In these columns we also report revised tranche ratings and we calculate by how many notches the tranches are downgraded.

		Standard market conditions (PD = 10%)			Deteriorated market conditions (PD=13%)		
Tranche	Tranche subordination CDO/CDO-sq.	Default probability			Default probability		
		Bond	CDO	CDO-squared	Bond	CDO	CDO-squared
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tranche 1 equity	0% / 0%		98.33% 'NR'	77.88% 'CCC-'		99.34% 'NR'	92.23% 'NR'
tranche 2 junior mezz.	9.90% / 13.27%	10.00% 'BBB-'	10.00% 'BBB-'	10.00% 'BBB-'	13.00% 'BB+' 1 notch	19.14% 'BB-' 3 notches	29.83% 'B+' 4 notches
tranche 3 senior mezz.	14.75% / 24.92%		1.97% 'A-'	2.07% 'A-'		4.96% 'BBB' 2 notches	10.15% 'BBB-' 3 notches
tranche 4 senior	17.08% / 31.25%		0.87% 'AA'	0.87% 'AA'		2.40% 'A-' 4 notches	5.46% 'BBB' 6 notches
tranche 5 super-senior	19.45% / 37.50%		0.36% 'AAA'	0.36% 'AAA'		1.06% 'AA-' 3 notches	2.87% 'BBB+' 7 notches

credit conditions. For illustration, we calculate the conditional default probabilities of the surviving obligors given that the first default in the collateral portfolio occurs at time t. This analysis is based on the CDO collateral portfolio of 100 bonds whose detailed description along with assumptions is presented in Section 3.[27] The results are plotted in Figure 3.

Figure 3 shows that a sudden and early default event can lead to a large increase in default probabilities of the surviving names. If the first default occurs after 1, 3 and 6 months, then the conditional default probabilities of the surviving bonds jump to, respectively, 16.19%, 13.31%, 11.14%. We thus see that a

[27] Appendix B explains how the conditional default probabilities are calculated.

deterioration in credit conditions considered in Table 4 corresponds to a single default after roughly 3 months, which is not an unlikely scenario under stressful credit conditions. If the first default occurs exactly after 1 year, then the conditional default probabilities of the surviving bonds equal 8.86%, which roughly corresponds to the unconditional default probability of 1% per annum over the remaining 9 years until maturity. That is because the first default occurring after 1 year is neutral information for market participants as it is in line with ex-ante expectations about credit quality.

Related to the problem of default contagion is the fact that the rating agencies derive default probability estimates of collateral bonds solely based on their credit ratings. It means that these are 'through-the-cycle' estimates reflecting (average) historical default frequencies of similarly-rated bonds. During economic recessions, therefore, the actual default probabilities of collateral bonds can be expected to significantly exceed the rating agencies' estimates. That is particularly important because collateral portfolios typically consist of bonds with low investment-grade ratings (such as BBB), which are known to perform pro-cyclically with much higher default rates in economic recession.[28] As a result, a significant dete-

[28]An S&P definition of the BBB rating reads: "An obligor rated 'BBB' has adequate capacity to meet its financial commitments. However, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity of the obligor to meet its financial

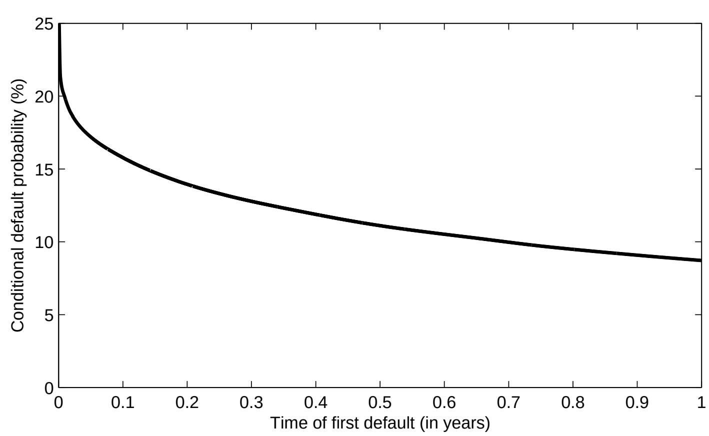


Figure 3: This figure plots default probability of the surviving bonds conditional on the information that the first default in the collateral pool occurs at time t. The range of t is up to one year after CDO issuance.

rioration in collateral credit quality (considered in Table 4) is most likely to occur when the economy enters a recession. CDO investors and the rating agencies should therefore strongly react to information about the point in the business cycle. This information is captured by macroeconomic indicators as well as the pace and severity of defaults within the universe of assets that are correlated with CDO collateral portfolios.

The rating agencies have to balance between the stability of ratings and ensuring that highly rated tranches do not incur losses. In contrast, investors in CDO tranches can continuously condition on current market developments and re-price CDO tranches according to market-implied default probabilities of the underlying bonds. The re-pricing of CDO tranches has critical implications for rating stability. During favorable market conditions, a CDO tranche is typically downgraded only when its credit enhancement is reduced due to realized defaults within the collateral portfolio. However, in deteriorating market conditions, a large fall in the market price of a tranche might force the rating agencies to take rating actions even prior to the realization of (significant) collateral losses. Due to reputational concerns, the rating agencies should timely revise tranche ratings to avoid situations when some tranche is still highly rated while its market value has already plummeted.

A decision to downgrade a tranche is made by a rating committee and this process is to some extent arbitrary. In particular, the rating agencies take into consideration market-wide factors. According to Fitch (2008) "The committees may make adjustment to standard assumption, or call for bespoke analysis. In addition, general economic outlook for certain sectors or industries may be taken into account". When the credit outlook is unfavorable and tranche prices are declining, the agencies might quote deteriorating market conditions and revise their rating assumptions. The severity of possible downgrading became apparent during the financial crisis when a large number of originally triple-A tranches were downgraded as far as to sub-investment grades. For example, S&P increased its estimate of the baseline correlation between RMBS tranches from 0.3 to 0.35-0.75, which resulted in massive downgrades of ABS CDOs (S&P, 2008).

In economically robust periods, highly rated CDO tranches as well as highly rated bonds tend to perform well. Tranches might be even characterized by higher rating stability because they have low exposure to idiosyncratic risks, but their capability to withstand economic recessions might be unsatisfactory. This fol-

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commitments". (S&P, 2009)

lows from the findings of Coval et al. (2009a) who show that default risks of CDO tranches are concentrated in systematically adverse economic states. Moreover, in case of a deterioration in credit conditions, the prices of CDO tranches are likely to become depressed more severely than the prices of corporate bonds. If this deterioration is prolonged relative to time until CDO maturity, then the prices of CDO tranches are unlikely to recover. Such risk properties of CDOs might go a long way in explaining the dramatic decline in prices of structured securities during the financial crisis.

In light of the above discussion, the comparability of rating and price stability between corporate bonds and CDO tranches is doubtful, particularly for highly rated tranches. In the corporate bond markets, the highest credit quality can be considered a guarantee of very low default risks and good rating stability. Notably, very few corporates qualify for the AAA rating, which in case of Fitch is about 1% of its total corporate coverage (Fitch, 2007). In contrast, almost 60% of the volume of CDO tranches rated by Fitch is assigned AAA ratings (Fitch, 2007). It seems that lower rating stability is the downside of the rating methodologies that allow for issuance of such large volumes of highly rated tranches, which in addition provide higher spreads relative to similarly-rated bonds.

7. Conclusions

This paper shows that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds implying that credit ratings are not sufficient for pricing. This creates rating arbitrage possibilities, which explains why structured securities can be so appealing to investors as well as issuers. Credit ratings reflect real-world payoff prospects, while pricing is done under the risk-neutral measure. On the one hand, CDO tranches are tailored to have sufficiently low default risks to meet the criteria for the highest credit ratings. On the other hand, these tranches must have sufficiently large risk-neutral expected losses to provide higher spreads relative to similarly-rated corporate bonds.

We have further examined risk properties of CDO tranches and we have shown that expected tranche payoffs are highly sensitive to default probabilities of the collateral bonds. This risk property is necessary for achieving the yield enhancement on tranches. However, the downside of such risk profile is that CDO tranches are inherently prone to incur large losses and massive downgrades when market conditions deteriorate. Tranche downgrading is exacerbated by default contagion and corresponding re-pricing of CDO tranches. It follows that CDO tranches have very different risk properties relative to similarly-rated corporate bonds.

Our findings show limitations of credit ratings understood as a universal measure of credit quality. Even if the rating agencies provide accurate and unbiased estimates of real-world default risks, then these estimates are still not very informative about the level of fair spreads. CDO tranches have inherently high risk premia, which are simply not captured by the rating methodologies. This suggests that improving the rating methodologies should go beyond minor changes such as better accounting for parameter uncertainty or using more conservative assumptions. The key issue to be considered is whether structured securities require a different rating approach that accounts for their distinguishing risk characteristics.

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Appendix A.

Table A.I: Comparison of fair spreads on tranches and risk-equivalent bonds under the assumption of 30% risk-neutral default probability of the underlying bonds.

In columns (1)-(3) we summarize tranche subordination levels and credit ratings. The structuring and rating is done under the baseline assumptions. In columns (4) and (5) we report tranche fair spreads, while column (6) reports fair spreads on the corresponding risk-equivalent bonds (separately for CDO / CDO-squared). To calculate fair spreads, we assume that risk-neutral default probabilities of bonds are triple the physical probabilities. The risk-equivalent bonds are defined as having the same (real-world) default probabilities and (real-world) expected losses as the corresponding tranches.

Tranche	Tranceh subordination CDO/CDO-squared	S&P / Moody's ratings	Fair spread (bps) CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	9.90% / 13.27%	'BBB-'/'Ba1'	784.92	1717.63	180.72 / 180.72
tranche 3 senior mezz.	14.75% / 24.92%	'A-'/'Baa1'	453.97	1407.68	42.49 / 42.75
tranche 4 senior	17.08% / 31.25%	'AA'/'A2'	306.75	1228.89	17.83 / 17.34
tranche 5 super-senior	19.45% / 37.50%	'AAA'/'AA1'	13.46	428.03	0.33 / 1.21

Table A.II: Comparison of fair spreads on tranches and risk-equivalent bonds under the assumption of 5% asset value correlation.

In columns (1)-(5) we summarize tranche subordination levels, ratings and fair spreads. The structuring and rating is done under the assumption that the asset value correlation is 5% instead of 12.5% used in the baseline case. In column (6) we report fair spreads on corporate bonds that are risk-equivalent to the corresponding tranches (separately for CDO / CDO-squared). The risk-equivalent bonds are defined as having the same (real-world) default probabilities and (real-world) expected losses as the corresponding tranches. To calculate fair spreads on these bonds, we assume that risk-neutral default probabilities are double the physical probabilities.

Tranche	Tranche subordination CDO/CDO-squared	S&P / Moody's ratings	Fair spread (bps)		
			CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	8.45% / 12.52%	'BBB-'/'BA1'	522.85	1271.46	111.95 / 111.95
tranche 3 senior mezz.	11.40% / 22.07%	'A-'/'Baa1'	282.38	1057.71	27.34 / 28.16
tranche 4 senior	12.64% / 27.35%	'AA'/'A2'	186.13	906.41	11.46 / 11.09
tranche 5 super-senior	13.96% / 33.41%	'AAA'/'Aa1'	4.15	237.00	0.12 / 0.62

Appendix B.

Consider a random vector $(V_1, V_2, ..., V_{100})$, which has a multivariate standard normal distribution with a common pair-wise correlation parameter ρ. Each V_i represents the scaled value of obligor i. We want to calculate default probability of the surviving obligors conditional on the information that the first obligor in the collateral pool defaults at a given time t. Without loss of generality (due to symmetry), we calculate the default probability of the first obligor conditional on the default of the 100th obligor at time t. In mathematical terms, this probability is given by:

$$Pr\left(V_1 \leq K | V_1 > k^*, V_2 > k^*, ..., V_{99} > k^*, V_{100} = k^*\right), \tag{B.1}$$

where K is the threshold corresponding to a default after 10 years and k^* is the threshold corresponding to a default at time t (e.g. 3 months).

A feature of the multivariate normal distribution is that the conditional distribution of $(V_1, V_2, ..., V_{99})$ given a known value of V_{100} is again normally distributed with adjusted conditional mean and variance matrices. The conditional mean of each V_i (corresponding to the surviving bonds) is ρk^*, the variance is $1 - \rho^2$ and the pair-wise correlations between V_i and V_j equal $\rho - \rho^2$. Using this conditional distribution, we can easily calculate the probability given by formula B.1 using simulations.

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