Pricing Contingent Convertibles: A Derivatives Approach

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Abstract

This article provides an in-depth analysis of pricing and structuring of contingent convertibles (CoCos). These debt instruments convert into the equity of the issuing bank or suffer a write-down of the face value upon the appearance of a trigger event. This trigger mechanism provides an automatic strengthening of the capital structure of the bank. Equity is in this case injected on the very moment the bank is failing to meet the minimum regulatory capital requirements or when it is heading towards a state of non-viability. In this paper the pricing of CoCos is handled using two different approaches. The first approach starts from a credit derivatives background. A second approach tackles the pricing and structuring of a CoCo as an equity derivatives problem. Both models are applied on the CoCos issued by LLoyds and Credit Suisse and allow to quantify the risks embedded within each of these structures.
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1 Introduction

1.1 Contingent Convertibles

Contingent convertibles made a very modest entry in the financial landscape in December 2009 when the Lloyds banking group offered the holders of some of its

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hybrid debt the possibility to swap their bonds into a new bond which carried a possible conversion into shares. Early February 2011 Credit Suisse did it more spectacular. This bank managed to attract easily $2 bn in new capital using this brand new asset class. Where Lloyds was initially struggling, Credit Suisse succeeded: investors gave this new issue a warm welcome which resulted in a massive oversubscription [28].
A Contingent convertible ("CoCo") is a debt instrument that automatically converts into equity or suffers a write down when the issuing bank gets into a state of a possible non-viability. This is a situation where the future of the bank is questioned by the depositors, bondholders and regulators. In order to quantify such a life threatening situation, the conversion or the write down is triggered by a particular pre-defined event. In this article, we will have a major focus on CoCos where a conversion in shares takes place. This automatic conversion makes this new product attractive from a regulatory point of view. The bank does not need to reach out to new investors in order to raise capital when it needs to reinforce its balance sheet. The conversion of debt in equity takes place automatically. No shareholders meeting is required. The concept of contingent capital fits perfectly in the concept of a more stable banking system. CoCos can be added to the list of other measures such as living wills, centralized clearing counterparties, higher capital requirements, lower leverage, higher liquidity ratios and other incentives that saw daylight in the aftermath of the 2008 credit crunch.
CoCos are not just another category of innovative hybrid debt which was conceived on some isolated trading desk. The whole idea of standby contingent capital is relatively old. We have to go all the way back to the United States, prior to banking act of 1933. The banking system was build on a system of a double liability. From 1850 to 1933, the risk taking in banks was constrained through this double liability system. Under this system all bank shareholders would be legally required, in the event of distress, for a down payment equal to the initial par value of the shares [14]. An initial share holder with an initial investment of $100 would be confronted with a sudden extra down payment of another $100. This is the very first concept of an unfunded contingent capital commitment.

1.2 Risk Profile

The risk profile of a CoCo is very similar to an insurance contract. An investor in this kind of debt, exposes himself to a limited upside but a large downside. The downside would occur when the investor has to deal with a triggering of the contingent convertible. This investor will in this case receive a pre-defined number of shares. This quantity is the conversion ratio (C_r). The share price on the moment the CoCo gets triggered will be low because all of this happens in a setting where the bank is going through a difficult period. How unlikely the probability of a trigger may be, the investor is going to deal with a loss when ending up with a conversion into cheap shares. The risk profile of a CoCo corresponds to an investment product with a low probability for a high loss and a high probability for a moderate gain. There always has been an appetite for this kind of risk in the markets. Institutional and retail investors embrace since 1998 reverse convertibles. One of the differences between CoCos and reverse convertibles is the fact that a Coco has not a fixed maturity: conversion can happen any time. More recently engineered structured products such as auto-callables carry a risk profile that corresponds even better to the risk embedded inside a CoCo. The popularity of auto-callables, which has globally spread to both European and U.S. investors ([1], [27]), might suggest that there is going to be enough appetite for contingent convertibles.

2 Anatomy

The fair value and the dynamics of a contingent convertible are driven by construction of the bond. The constructive elements that define the anatomy of the convertible are : conversion type, conversion fraction, trigger event and the conversion price. In this paragraph each of these constituents will be explained.

2.1 Trigger Event

The trigger specifies those circumstances where the bond will be converted into shares or where a write down will occur. This event is documented in the prospectus and lays out the setting from where a bank moves into difficult territory. After the trigger the bank ends up with a stronger capital structure. When structuring a CoCo the following conditions should be satisfied by the trigger event :

- **Clarity** The trigger has to carry the same message whatever the jurisdiction of the issuer. Suppose that an accounting ratio is used to define the moment from where the bond is converted into shares. If this particular accounting number is calculated using a different standard depending on the jurisdiction of the bank, then the trigger event will as a consequence have no clarity at all.

- **Objective** The process by which the life of a CoCo is ended and converted into shares should be known at the issue date. This has to be well documented in the prospectus. Ideally the triggering should require no external intervention at all. A CoCo, such as the one launched by Credit Suisse in February 2011, deviates from this guideline. Here, the swiss regulator has the right to step in and force a conversion into shares. This could be done before the injection of tax payers' money or if it considers the bank non-viable without writing off some debt and converting it into shares.

- **Transparant** A trigger defined as an event whereby the share price drops below a pre-defined barrier fits the test of transparency. Using a Core Tier 1 ratio to define the conversion is therefore sub-optimal. The investor has to rely on audited balance sheets that do not always provide enough details. Moreover these capital ratios are not forward looking, they are published with a delay.
- **Fixed** A trigger should be fixed and cannot be changed during the life of the contingent convertible.
- **Public** The trigger event or the data driving a possible conversion should be public information.

In total four different kind of trigger events can be defined: an accounting trigger, a market trigger, a multi-variate trigger and a regulatory trigger.

2.1.1 Accounting Trigger

In this category of trigger events, an accounting ratio is used as an objective indicator of the bank's solvability. The Lloyds and Credit Suisse CoCos for example, have been constructed with the Core Tier 1 ratio ($CT1$) as an indicator of the health of the bank. This ratio relates the Core Tier 1 capital against the risk weighted assets on the balance sheet. It is argued that any accounting number will only be triggered long after the facts [12]. The large US financial institutions that in 2008 either failed or had to be bailed out by the government, were reporting capital ratio's above the minimum requirement. Bear Stearns, Lehman Brothers, Wachovia and Merrill Lynch had regulatory capital ratio's far above the minimum level of 8%. This is one of the major objections held against the use of any accounting trigger in contingent debt. It views the viability of the bank from a point of view that can be distant from the economic reality. An accounting ratio such as a Core Tier 1 ratio is not continuously available. Unlike a share price there is no daily update and investors are kept in the dark about the state of the balance sheet. Only on a quarterly or semi annual basis, is this information released to the investor base. This leaves a lot of room for speculation regarding the fact whether the bond will be triggered or not. A trigger system founded on one accounting number is easy to game [2]. One could therefore consider a mix of balance sheet information to be used in the examination of the health of the financial institution [24].

2.1.2 Market Trigger

This is the favorite trigger mechanism for the academic world. A variable such as a share price or CDS spread is the corner stone of this trigger mechanism. A share price is a forward looking parameter. It translates the view of the market on the fate of the bank. When the share price breaches a well defined barrier level, this is considered as the sign to convert the bond into shares. Using a market based trigger comes at a risk however. An investor holding contingent convertible debt exposes his portfolio to risk that the bonds get converted because of market manipulation. Just imagine that another market player sells a large amount of shares when the share price is already trading very close to the barrier. If such a transaction takes place on a day with no volume, the CoCo might trigger. The flash crash that took place in the afternoon of May 6, 2010 in the United States, illustrates this. Almost all of the 8000 securities traded in the U.S. suffered an aggressive price correction . Over 20.000 trades across more than 300 securities were executed at price which was more than 60% away from their values just before. There would be blood on the street for contingent debt holders.

2.1.3 Regulatory Trigger

In the bail-in capital solution, regulatory triggers are the core of the system. The government decides when to convert the bonds into shares or when to write down a fraction of the face value. The Basel Committee's proposal to use a debt write-down or equity conversion provision in the terms and conditions of new capital instruments puts an emphasis on the use of such a regulatory trigger. The decision of a national regulator that a bank has become non-viable, would trigger the conversion into shares or the write-down of debt [19]. The presence of such a regulatory trigger could reduce the marketability of a bond. For some investors it feels indeed like handing over a blank cheque to the government. The use of these kind of triggers would furthermore also be a difficult hurdle to take when pricing these securities. Quantifying the expected behavior of the regulator is indeed an impossible task.

2.1.4 Multi-Variate Trigger

One could increase the dimensionality in the triggering process of a CoCo. Instead of using one single metric specifying a conversion, one could combine several triggers into one multi-variate trigger. One component of this multi-variate trigger could focus on the underlying company and combine this with a universal systemic trigger that acts as a gauge for the overall state of the economy. The Squam Lake Working Group on Financial Regulation [20] has a proposal in this direction. They advocate to use such a multi variate trigger. Here, the regulator would in a first step declare a state of emergency. Next to this macro trigger there is a micro trigger, that needs to be fulfilled to have a conversion into shares. The micro trigger corresponds for example to a capital ratio falling below a pre-set standard. This dual trigger would ensure a recapitalization of problematic banks in a situation whenever the financial industry is facing tough times.

2.2 Conversion Fraction (α)

The conversion amount is that portion of the face value N that can be converted or written down. The conversion amount is specified as αN where α is the conversion fraction. When $\alpha = 1$, we are dealing with a "full" CoCo. The shadow committee of the American Enterprise Institute, made some recommendations on the way Coco's should be structured [10]. One of these recommendations is that the full face value of the contingent convertible should be converted into equity ($\alpha = 1$). Converting just enough shares to meet the regulatory capital would not restore confidence of the market in the bank. This argument is not followed everywhere and some would favor the conversion of just enough bonds to make the financial institution solid but no more than that. This is point of view corresponds to a CoCo structured with $\alpha < 1$.

2.3 Conversion Type

The CoCo can convert into a predefined number of shares. A second possibility is that the face value of the debt is written down. The number of shares received per converted bond is the conversion ratio C_r. The implied purchase price of the underlying shares is given by the conversion amount divided by this conversion ratio. This is the conversion price:

$$C_p = \frac{\alpha N}{C_r} \tag{1}$$

2.4 Conversion Price (C_p)

A CoCo investor is better off when the conversion price is low. This leads to a high number of shares received upon conversion. For the current shareholders of the bank, the opposite is true. A low conversion price leads to a high dilution of their equity investment. The level of the conversion price has an important impact on the dynamic behavior of the contingent convertible and its fair value. The extend to which the existing shareholder gets diluted depends on the conversion mechanism used. A low conversion price increases the equity sensitivity of the instrument in a situation where the CoCo is about to be triggered. In practice three different conversion price choices are possible:

- **Conversion Price = Price on Trigger**
  The conversion price is set equal to S^*, this is the share price observed on the trigger moment T^*. Since triggering only happens when the fate of the bank is not really looking rosy, this will be a low share price. Going for a conversion price equal to the market price of the share when the trigger happens, or picking an average of those share prices observed during a short period after this trigger, is a choice that gives the investor a high conversion ratio. The share holders will have to accept a serious watering down of their share holdings.

- **Conversion Price = Price on Issue**
  This choice is on the other side of the spectrum. In this particular case $C_p = S_0$ where S_0 is the share price of the bank on the issue date of the contingent convertible. This conversion price typically will bring about a low conversion ratio. It might be the preferred choice for the existing share holders because it will not cause a lot of dilution on their current equity investment in the bank.

- **Conversion Price with a Floor**
  This conversion price offer a compromise between the two previous solutions. The share price is set equal to the price on the moment the bond gets triggered into conversion, but is not allowed to drop below a certain value S_F. Using such a floor, the conversion price becomes:

$$C_p = \max(S^*, S_F) \tag{2}$$

A consensus in the CoCo-debate regarding the best mechanism is still going on. Lloyds opted for $C_p = S_0$ whereas Credit Suisse used a conversion price with a floor. Regulators have not given any indication so far regarding the level of the conversion price. The market forces will go for the best design.

2.5 Funded or Unfunded

The few CoCo examples we covered so far, fall under the category of funded financial instruments. The issuer receives an upfront payment by the investor, there are no further installments required. Upon the trigger event the capital structure of the bank's balance sheet gets reinforced, but there is no injection of cash. A cash injection was done on the issue date. An example of an unfunded contingent convertible, is the solution engineered by the UK government for the benefit of Royal Bank of Scotland in 2010. In this structure the taxpayer would step in and provide capital as soon as the Core Tier 1 ratio of RBS drops below 5%. The trigger of the contingent capital facility will strengthen the bank that got triggered, but could weaken the institution that underwrote the facility. An unfunded contingent capital solution carries therefore a lot of counterparty risk. In the RBS case the counterparty is the UK government, which is from a default perspective less of an issue [24].

2.6 Difference with convertible bonds

The contingent convertibles issued by Lloyds and Credit Suisse convert into shares upon the arrival of a trigger event. The conversion into shares is a property which CoCos share with the traditional convertible bonds. However, this does not allow the investor to label these instruments as another convertible bond variety. CoCos are a totally different asset class. They offer nor a limited downside nor an unlimited upside. On the contrary, the upside potential of a contingent convertible is limited whereas the investor will be exposed to the full downside once the bond converts into shares. The fact that some traditional convertible bonds are issued with a so-called "CoCo-feature" unfortunately adds to the confusion. This feature only points to the fact that the convertibility of the bond is contingent on the level of the share price [16]. This blurs the boundary between convertible bonds and CoCos. Was this confusion the reason for the fact that this brand new asset class received initially only a luke warm response in the financial market? The fact that a brand new concept such as a CoCo saw daylight in the midst of some of the largest regulatory overhauls ever, was not helpful either.

Figure 1 illustrates the difference in price behavior between a convertible bond and a contingent convertible. In this figure we compare a convertible and a CoCo issued by the same financial institution. To simplify the matter, both instruments in this example share the same underlying share, the same maturity and have an identical conversion price. The only difference is the presence of a market trigger for the Coco. The value of the convertible bond converges to a bond floor in the case of falling share prices [23]. This is the present value of the cash flows embedded within the convertible. This lower boundary of the convertible price acts at the same time as an upper boundary for the CoCo. The CoCo has indeed a limited upside.

In this particular example we are dealing with a CoCo where the conversion into shares happens as soon as the share price S drops below a market trigger S^*. From this moment onwards, the CoCo investor receives shares and becomes an equity investor. This holding of C_r shares is represented by the parity line in

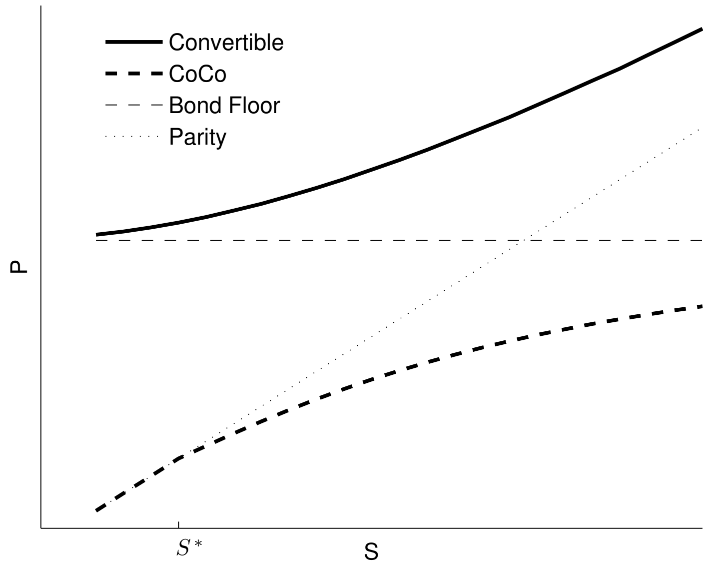

Figure 1: Price graph of a convertible bond and a CoCo with a trigger S^*

Figure 1. The value of the convertible bond converges to parity for an increasing share price, the opposite is true for the CoCo.

2.7 CoCo examples

The engineering of contingent capital notes is clearly a project which is still under construction. Cocos were the subject of many conferences and newspaper articles. The academic world enthusiastically welcomed this topic. Three banks decided not to wait and went ahead with their issue of a contingent convertible. A summary of each of the bonds can be found in Table 1.

- The **LLoyds (ECN)**
  The enhanced capital notes issued by the LLoyds Banking Group offered existing owners of hybrid capital the possibility to exchange some existing hybrid instruments into a contingent convertible. LLoyds had received state aid from the UK government and the acceptance of this tax payers money would not allow holders of hybrid capital to receive coupons. The only way for an investor to keep receiving a steady income stream was to swap these poor performing hybrid bonds into one of the new series of CoCos. These CoCos, issued in November 2009, benefited from a high coupon but

Issuer	Lloyds	Credit Suisse	Rabobank
Full Name	Enchanced Capital Notes	Buffer Capital Notes	Senior Contingent Notes
Abbreviation	ECN	BCN	SCN
Issue Size	GBP 7 bn (32 series)	USD 2 bn	EUR 1.25 bn
Issue Type	Exchange	New Issue	New Issue
Rating	BB	BBB+	-
Issue Date	December 1, 2009	February 17, 2011	March 12, 2010
Subordination	Lower Tier 2	Tier 2	Senior
Maturity	10 - 20 year	30 year - callable after 5 year and 6 months	10 year
Coupon	1.5 - 2.5% increase of the coupon of the hybrid bond exchanged for the ECN	7.875%	Libor + 3.5%
Coupon Deferral	No	No	No
Trigger Contingency	Conversion into a fixed number of ordinary shares	Conversion into a fixed number of ordinary shares	Write down with a 25% cash recovery
Conversion Price	59 Pence	$\max(\text{USD}20, \text{CHF}20, \overline{S})$	
Trigger Type	Accounting	Accounting and Regulatory	Accounting
Accounting Trigger	Core Tier 1 Ratio	Core Tier 1 Ratio	Equity Capital/RWA
Accounting Trigger Level	5%	7%	7%
Regulatory Trigger		The Swiss regulator determines that the Credit Suisse Group requires public sector support to prevent it from becoming insolvent	

Table 1: Overview of three contingent convertible bonds. $\overline{S}$ is the weighted average price of the Credit Suisse shares in a 30 day period prior to conversion.

introduced also the risk of being converted into equity as soon as the bank's Core Tier 1 capital falls below the trigger level of 5%. The early investors in this first Coco issue ever, were for 30% hedge fund investors [7].

- **Rabobank (SCN)**
  Rabobank used a different version of contingent capital in early 2010. This time the trigger would not result in a conversion but in a write down of 75% of the par amount. Rabobank was not a listed company and hence a write down is more natural than a conversion into equity.

- **Credit Suisse (BCN)**
  Credit Suisse launched two CoCo bonds early 2011. The first issue was offered to two strategic holders of hybrid Tier 1 debt issued back in 2008. The Qatar Holding LLC[1] and the Olayan Group[2] were offered to exchange these notes, no longer qualifying as Tier 1 debt under Basel III, in exchange for a new contingent convertible. A few days later, Credit Suisse came with a public issue. This second issue attracted as much as $2bn in new capital. Investors gave this bond a warm welcome. [28].

2.8 CoCo-Note Variations

Contingent capital is work in progress and in 2010 there was already one variation on this theme. Unicredito and Intesa Sanpaolo, two Italian banks, issued a bond where the face value would be written down as soon as the total capital ratio falls below a trigger level of 6%. The bonds are **Write Down /Write Up** bonds because the process can be reversed if the financial health of the bank would improve.

Bonuses paid in the financial industry, were the topic of a lively debate in 2009 and have dominated the headlines in the financial press. After using tax payers' money to save banks, governments decided to get involved and imposed higher taxes on these payments. Financial institutions were encouraged to use deferrals in their cash bonus payments going forward. Some banks consider an alternative route and will distribute bonuses structured as CoCo's. The amount of these **CoCo bonuses** held by the employees will be too small to have a significant impact on the stability of the bank when a trigger happens, but it aligns the interests of the bank and its employees. Risk taking should not be encouraged by a bonus policy [9]. One of the first banks to probably go ahead with a CoCo bonus scheme is

[1]The Qatar Holding LLC is based in Doha, Qatar. It operates since 2006 as a subsidiary of the Qatar Investment Authority.

[2]The Olayan Group from Saudi Arabia is a private multinational enterprise founded in 1947 by the Olayan family

Barclays [26].
The latest novelty is a **CoCoCo** and is a combination of a CoCo and a convertible bond. In bad times, the bond gets triggered and the bond holder becomes an equity investor after receiving cheap shares. This is the CoCo as we know it, an instrument with a large possible downside and limited upside. A CoCoCo deals with this limited upside and allows an investor to convert the bond into shares. This twist gives more upside to the bond holder. The bank of Cyprus plans to issue up to €1.4bn of CoCoCos [17]. These notes would have an accounting trigger set at $CT1 = 5\%$ but allow the investor to convert into shares on a quarterly basis.

2.9 The Market for CoCos

The $1trillion of CoCo issuance which is expected to hit the market according to Standard and Poor's [8], will need to be absorbed by a new investor base. The success of the contingent convertibles depends on how broad the investor base is going to be. If the buying and selling of CoCos remains constrained to banks, that are buying each other issues, then the "too big too fail" problem remains unchanged. A CoCo triggered in one bank, will hit the asset pool of the bank that invested in these bonds. The interconnectedness in the financial industry will only get worse.
In order to get other investors on board, some hurdles have to be taken. This new asset class already got the full sympathy from the regulators. Canada and especially Switzerland took a positive stance for example. The Swiss were ahead on the curve when allowing contingent debt to be issued in order to increase the capital base of its largest banks, more specifically Credit Suisse and UBS [3]. Another pillar on which the possible success of CoCos will depend, is the way credit rating agencies are going rate this type of asset class. These organizations could argue that contingent debt cannot be rated because of the conversion probability. If this type of debt does not get rated, this will reduce the appetite of some institutional investors. Very often the mandates under which these investors operate would typically only allow debt instruments with an investment grade rating. In November 2010, Fitch was the first rating agency to make its intention to rate contingent debt public [5]. Traditionally where one agency goes, others will follow in its footsteps and start rating contingent capital as well.
The inclusion in a benchmark index such as those compiled by Bank of America Merrill Lynch and Barclays Capital, would also be a guarantee for a successful CoCo issuance. Investors who use these indices as benchmarks, will otherwise not invest in contingent debt. The Barclays Capital Index Product Group has made it clear however [13], that contingent capital is - similar to other mandatory convertibles - not eligible to be included in the broad-based investment grade Barclays Capital Bond indices.

3 Pricing

3.1 Introduction

In this paper we present different ways to tackle the pricing of contingent convertibles. The existence of different valuation methods should not be a surprise because a CoCo is a hybrid security sitting between equity and debt on the balance sheet of the bank. A CoCo valuation model can find its roots both in equity derivative or fixed income mathematics. Starting from the viewpoint of a fixed income investor, we can construct a **credit derivative pricing model**. For fixed income investors, the CoCo pricing problem boils down to the extra yield needed on top of the risk free rate in order to accept the risk of facing a loss. This loss would materialize when bond triggers into shares. An equity derivative specialist will see the contingent convertible bond as a long position in C_r shares that are knocked in one a trigger event materializes. Using barrier options one can follow an **equity derivatives approach** to price contingent debt.
A third alternative pricing model is a **structural model**. Under the structural philosophy one needs to model the assets on the balance sheet and develop a stochastic process for these. Starting from the stochastic model for the assets, all the other capital instruments are going to be derived. After all, the total value of the assets held by the bank is equal to the sum of all the capital instruments. The book value of the equity, the senior bonds, deposits and CoCos should all add up to the value of the assets. The structural model takes the interaction between each of these components into account. An example of a structural model for the pricing of contingent capital can be found in [21] and [6]. In [18], the authors elaborated on the use of conic finance in the valuation of contingent convertibles.

3.2 Credit Derivatives Approach

3.2.1 Introduction

In a reduced form approach, a default intensity parameter λ is used when modeling default. The probability that a financial institution, that issued a bond, goes default in the time interval $[t, t + dt]$ while surviving up to the time t is equal to λdt. Accordingly one can show that the probability that the bond survives the next T years, is given by $\exp(-\lambda T)$. This is the survival probability p_s. The default probability over the same horizon is hence $1 - \exp(-\lambda T)$. This theory forms the basis of the **intensity based credit modeling** or the **reduced form approach**. Given a value of λ we can calculate the survival probabilities and price corporate bonds accordingly. The reduced form method took credit derivatives pricing by storm and is extensively covered in [11]. On default the investor expects to recover a portion of the face value N of the bond. This is the recovery rate R. On default the loss for the investor is equal to $(1 - R) \times N$. The relationship between the credit spread (cs), recovery rate and default intensity is given by the following equation [23]:

$$cs = (1 - R) \times \lambda \tag{3}$$

This equation allows to move in approximative way from the knowledge of a recovery rate and a default intensity to a credit spread cs. This credit spread is the product of the percentage loss $(1 - R)$ and the probability of the loss taking place (λ). We could now model the trigger event where a Coco is converted into shares, as an extreme event similar to the way default is statistically modeled in corporate debt or in credit default swaps. Hitting the trigger would be modeled as some kind of a special case of a "default" event. The default intensity λ will accordingly be substituted by a trigger intensity λ_{Trigger}. By construction, it will be more likely that the contingent note is forced into a conversion then that the bank will default on its outstanding corporate bonds. The CoCo triggers before default takes place. Hence :

$$\lambda_{\text{Trigger}} > \lambda \tag{4}$$

Using Equation 3, we can determine the value of the credit spread on contingent debt, using the following rule of thumb:

$$cs_{CoCo} = (1 - R_{CoCo}) \times \lambda_{\text{Trigger}} \tag{5}$$

3.2.2 Loss

The loss of the contingent convertible when a trigger happens is driven by the choice of the conversion price. This is illustrated in Figure 2 and holds for any kind of trigger:

$$Loss_{CoCo} = N - C_r S^* = N\left(1 - \frac{S^*}{C_p}\right) = N(1 - R_{CoCo}) \tag{6}$$

The share price on the moment the bond is converted into shares is S^*. The recovery rate on the triggering of the convertible is hence given by the ratio of this share price and the conversion price C_p:

$$R_{CoCo} = \frac{S^*}{C_p} \tag{7}$$

This illustrates the impact of the conversion price on the value of the CoCo. If the conversion price is set equal to the share price observed on the moment of

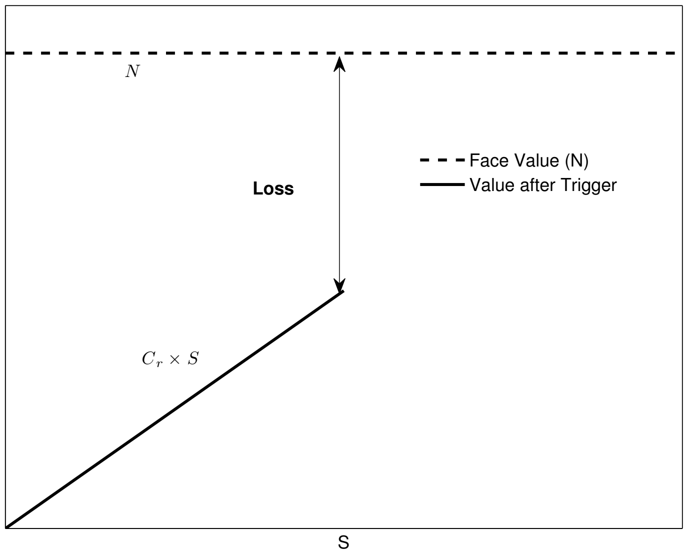

Figure 2: Loss on a contingent convertible when the trigger is hit

the trigger, there is no loss for the investor. The investor receives shares because of the trigger, but the total value of these shares is equal to the face value N. The choice of C_p and the estimation of S^* allows to calculate the loss component in Equation 3. This is the same starting point as the model described in [15]. The only missing link is the trigger intensity λ_{Trigger}.

3.2.3 Trigger Intensity (λ_{Trigger})

The probability that a trigger occurs in the time interval $[t, t+dt]$ while not being triggered up t is given by $\lambda_{\text{Trigger}}dt$. A trigger could for example be defined as a regulator declaring the bank non- viable without government support, a market parameter such as a stock price dropping below a pre-defined barrier or the announcement by the bank it has insufficient Core Tier 1 capital. Modeling regulatory behavior is an impossible task. The same difficult challenge holds when trying to engineer a stochastic model for an accounting measure based on capital ratios. In stead of directly modeling an accounting or a regulatory trigger, we could associate with these events a corresponding market trigger. An accounting trigger where a core Tier 1 ratio drops below a minimum level, could be replaced by an equivalent event where the stock price drops below a barrier S^*. Linking a market trigger S^* to an accounting trigger is illustrated in Figure 3.

The probability p^* that such a level is touched during the life T of the contingent

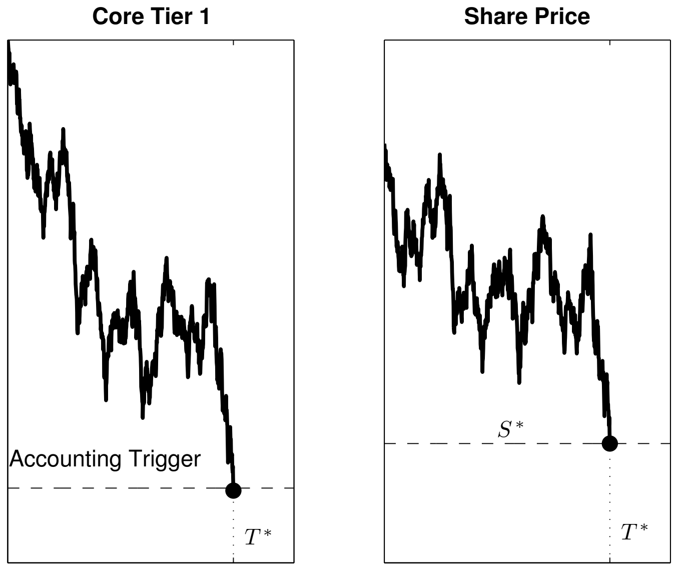

Figure 3: Trigger of an accounting trigger is associated with a market trigger on the share price.

convertible is given by the following equation [25]:

$$\begin{array}{lcl} p^* & = & N(\frac{\log(\frac{S^*}{S})-\mu T}{\sigma\sqrt{T}}) + (\frac{S^*}{S})^{\frac{2\mu}{\sigma^2}} N(\frac{\log(\frac{S^*}{S})+\mu T}{\sigma\sqrt{T}}) \\ \mu & = & r - q - \frac{\sigma^2}{2} \\ q & : & \text{Continuous dividend yield} \\ r & : & \text{Continuous interest rate} \\ \sigma & : & \text{Volatility} \\ T & : & \text{Maturity of the contingent convertible} \\ S & : & \text{Current share price} \end{array} \tag{8}$$

This is the first exit time equation used in barrier option pricing under Black-Scholes. It models the probability that a stock price S will touch the level S^* somewhere between today at the expiration of the bond T years from now. $N(x)$ is the probability that a random variable X, which is following a standard normal distribution, takes a value smaller than x:

$$N(x) = \text{Probability}(X \leq x) \tag{9}$$

In this equation p^* quantifies the probability that the trigger is happening. From p^* we can now determine $\lambda_{Trigger}$:

$$\lambda_{Trigger} = -\frac{\log(1-p^*)}{T} \tag{10}$$

This subsequently gives the Coco spread (cs_{CoCo})[3] :

$$cs_{CoCo} = -\frac{\log(1-p^*)}{T} \times \left(1 - \frac{S^*}{C_p}\right) \tag{11}$$

3.2.4 CoCo Spread Calculation Example

Suppose we are dealing with a newly issued contingent convertible. The only trigger in this example, is a regulatory trigger. The Coco has a 10 year maturity. The underlying share is price set at \$100, has a volatility (σ) equal to 30% and is expected not to distribute any dividend at all ($q = 0$). The continuous interest rate (r) is 4%. We now assume that the occurrence of the trigger corresponds to a share price equal to half the current share price ($S^* = \$50$). The calculation of the credit spread under this assumption is given by the following three step process:

1. Probability of hitting the trigger (Equation 8)
   $p^* = 48.30\%$

2. Trigger intensity (Equation 10)
   $\lambda_{Trigger} = -\frac{\log(1-0.4830)}{10} = 0.066{=}6.6\%$

3. Recovery (Equation 7)
   $R_{CoCo} = 1 - \frac{S^*}{C_p} = 1 - \frac{50}{100} = 50\%$

Hence : cs_{CoCo} **= 6.6%** $\times$ **50% = 330 bps**. Adding this spread to the continuous interest rate, gives the total yield on this Coco : 7.30%.

3.2.5 Coco Delta

The yield of 7.3% obtained in the previous example is a function of the stock price. A deterioration in the level of the share price S would bring about an increase in cs_{CoCo} and a decrease in the CoCo price P:

$$\frac{\partial cs_{CoCo}}{\partial S} < 0 \tag{12}$$

The Coco has a positive Delta:

$$\Delta = \frac{\partial P}{\partial S} > 0 \tag{13}$$

[3] This spread is defined as a continuous interest rate and needs be scaled back to the yield and day count convention of the bond

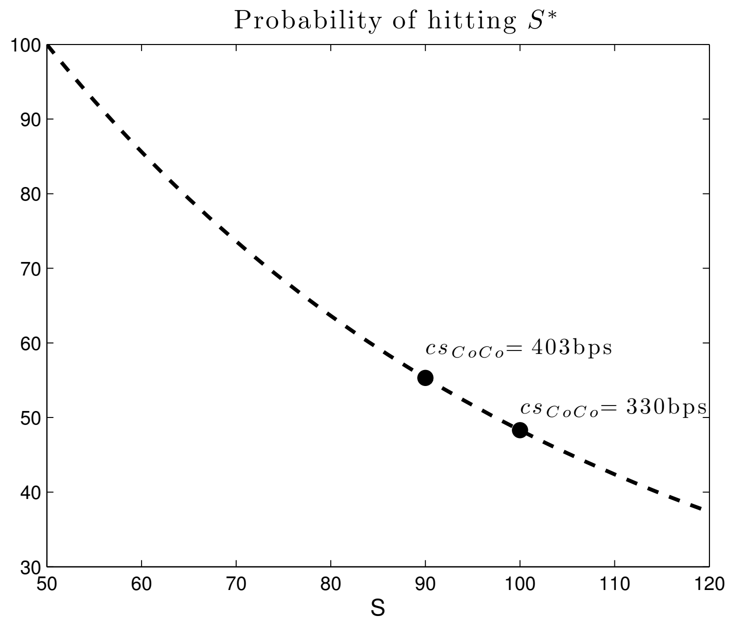

Figure 4: Probability p^* of hitting the trigger level S^* for different share prices.

Assume for example that the share price level around which the CoCo is expected to get triggered does not change, but that S decreases with 10%. from \$100 to \$90. A conversion is more likely since S is now closer to S^*. The investors will therefore demand a higher yield from the contingent convertible. In this case the CoCo spread moves from 330 to 403 bps. A graphical presentation illustrating the sensitivity of cs_{Coco} is given in Figure 4.

3.2.6 Case Study : Credit Suisse

The Credit Suisse contingent convertible (Buffer Capital Notes) issued in February 2011, has a different conversion mechanism than the ECNs from the Lloyds Banking Group. The conversion price is set at the trigger date $C_p = S^*$, not at the issue date. This could lead to an unlimited dilution for the shareholders and in order prevent this from happening, a floor is applied on this conversion price. Such a floor effectively puts a maximum on the number of shares created when the Coco get triggered:

$$C_p = \max(USD20, CHF20, S^*) \tag{14}$$

The value of S^* is fixed by taking the weighted average share price of Credit Suisse in a 30 day period preceding the conversion. Equation 11 now becomes:

$$cs_{CoCo} = -\frac{\log(1-p^*)}{T} \times (1 - \frac{S^*}{\max(USD20, CHF20, S^*)}) \qquad (15)$$

The closing price of Credit Suisse on March 21, 2011 equals \$42.84 and the credit spread on the BCN is 488 bps. We can determine the level of the market trigger S^* implied by the quoted spread on the CoCo. The other pricing parameters are:

- $r = 2.42\%$: 5 year USD interest rate
- $q = 3\%$: Dividend Yield
- $\sigma = 31\%$: Volatility

The Coco is priced up till the first call date 5.5 years from the pricing date.The results of this exercise are represented in Figure 5. The implied trigger price corresponding to a spread of 488 bps, is equal to \$15.68. This is corresponds to a level equal to 36.6% of the closing price of Credit Suisse on March 21, 2011. This trigger takes place below the floor on the conversion price (\$20). The coco holder is suffering a loss equal to 21.6% ($= 1 - \frac{15.68}{20.00}$).

3.3 Equity Derivative Approach

3.3.1 Introduction

Similar to the credit derivative approach, only the pricing of contingent debt that is converted into shares on the appearance of a trigger event, will be studied. The Coco's with a write down of the debt once a trigger materializes, are not going to be our main point of interest. The contingent convertibles issued by Lloyds and the developments in Switzerland regarding Credit Suisse and UBS, are indicating that the convertibility into shares will possibly be the main Coco model going forward.

3.3.2 Zero Coupon Coco

A zero coupon contingent convertible has a face value N, which will be paid out at maturity T. The bond distributes no coupons and the conversion into shares will be forced upon the investors once the trigger is a fact. At the trigger moment a certain fraction of the face value αN is up for conversion. The remainder $(1-\alpha)N$ will be redeemed in cash as set out in the prospectus. The final payoff of the Coco is P_T and is given by the following equation :

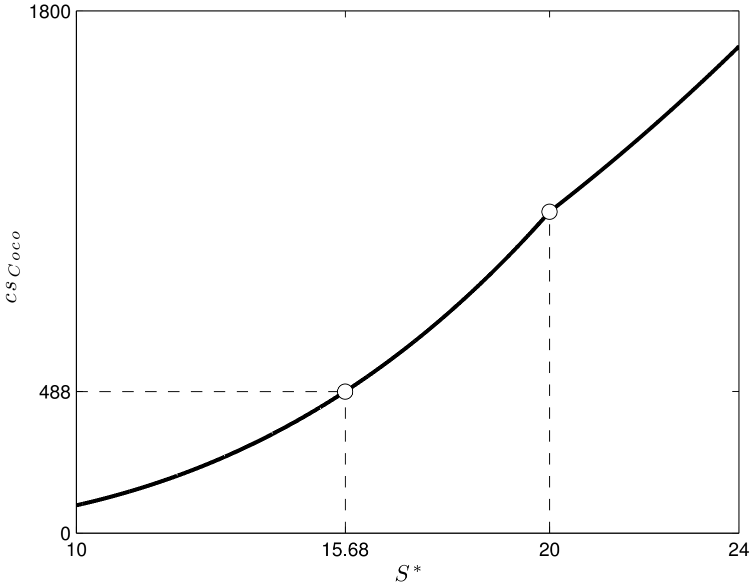

Figure 5: For different expected trigger levels S^*, the corresponding credit spread of the contingent convertible has been calculated for the BCN of Credit Suisse. The implied trigger level corresponding to a credit spread of 488 bps is \$15.68.

$$\begin{cases} P_T = (1-\alpha)N + C_r S_T & \text{if triggered} \\ P_T = N & \text{if not triggered} \end{cases} \tag{16}$$

The trigger indicator $\mathbf{1}_{\{\text{Trigger}\}}$ equals 1 when the Coco is triggered and is zero otherwise. After such a trigger the bond holder is long C_r shares and will receive $(1-\alpha)N$ at maturity. The amount $(1-\alpha)N$ is the unconverted fraction of the Coco. When dealing with a market based trigger where the share price has to fall below a pre-defined barrier S^*, this indicator is written as:

$$\mathbf{1}_{\{\min (S_t)_{0 \le t \le T} \le S^*\}} \tag{17}$$

Dealing with a contingent convertible where the Core Tier 1 ratio needs to be at any time above 5% in order to avoid a conversion into shares, the indicator on this accounting trigger is defined as :

$$\mathbf{1}_{\{\min (CT1)_{0 \le t \le T} \le 5\%\}} \tag{18}$$

Equation 16 can be rewritten as :

$$\begin{aligned} P_T &= N + C_r \times (S_T - \tfrac{\alpha N}{C_r})\mathbf{1}_{\{\text{Trigger}\}} \\ &= N + C_r \times (S_T - C_p)\mathbf{1}_{\{\text{Trigger}\}} \end{aligned} \tag{19}$$

One can observe in Equation 19 how the final payoff of a Coco can be broken down into two components. There is first of all the face value of the bond N next to a possible purchase of C_r shares. This second component only materializes if a trigger actually took place during the life of the bond. We approximate this by a knock-in forward on the underlying shares. This models a possible purchase of C_r shares. The purchase price of each share is the conversion price C_p. A Coco is a combined position of a (zero coupon) corporate bond and a knock-in forward (F) on Cr shares of the issuer :

Zero Coupon Coco = Zero Coupon Corporate Bond + Knock-In Forward(s)

The knock-in forward is priced as a long position of a knock-in call and a short position in a knock-in put. Both options share the same strike C_p and the same barrier S^*. The decomposition of a Coco into corporate debt and C_r knock-in forwards on shares, allows us to price contingent convertibles using a closed form formula. The elegance of the pricing model comes at a cost however. The approach chosen introduces a flaw in the model. In reality the investor receives shares and not forwards on the trigger event. The conversion value on the trigger moment is $C_r S$ and not $C_r F$. The difference is outspoken if the trigger event takes place a long time before the final expiration date of the bond. Being long shares or a forward on those shares carries a consequence if dividends would be paid out for example. The investor holding shares obtained through a conversion of the Coco, is immediately entitled to dividends and voting rights. Under the acceptable assumption that the dividend payout after a trigger took place is going to be low, the barrier option technique is an acceptable model. A bank will indeed not start paying out large dividends to the holders of its common equity, when it just recapitalized its balance sheet. The value of the zero-coupon Coco on the valuation date P_t is then:

$$P_t = N \exp(-rT) + \text{Knock-In Forward} \tag{20}$$

The corporate bond part in Equation 20 is calculated using the risk free rate. The loss on conversion - which is some to some extend a default event - is embedded in the knock-in forward part of the pricing model.

3.3.3 Adding Coupons

The coupons of the Coco will only be received as long as the trigger event is not a fact. This reduces the value of a Coco compared to a straight corporate bond of the same issuer. Suppose the bond pays out k coupons with value c_i at times t_i. When a conversion trigger occurs, the coupon c_i is reduced to : $(1-\alpha)c_i$. This claim reduces the price of the Coco and needs to be valued as a short position in a binary down-and-in option (BDI). For each coupon c_i, there is indeed a corresponding short position in binary option that is knocked in on the barrier. Each of the k binary options BDI comes with a negative rebate of $-\alpha c_i N$ hereby offsetting partially or completely ($\alpha = 1$) the coupon expected at time t_i. This will be the case whenever there is a trigger before the coupon date t_i. A knock-in effectively eliminates the coupon.

Coco = Corporate Bond
+ Knock-In Forward(s)
- $\sum$ Binary Down-In Options

For every coupon c_i there is a matching BDI option with a maturity corresponding to the maturity date t_i of each of the coupons, and a trigger event which matches the overall conversion trigger of the Coco. The sum of these BDI options lowers the price of the Coco:

$$-\sum_{i=1}^{k} \alpha c_i \exp(-rt_i) \mathbf{1}_{\{\text{Trigger Time} \leq t_i\}} \tag{21}$$

The price P of the CoCo is equal to a corporate bond (**A** in Equation 22) to which a knock-in forward is added (**B**) . The third component is the sum of the BDI options which offset the coupons c_i upon the occurrence of a trigger. (**C**) .

$$
\begin{aligned}
\mathbf{P} &= \mathbf{A}+\mathbf{B}+\mathbf{C} \\
\mathbf{A} &= N\exp(-rT)+\textstyle\sum_{i=1}^{k} c_i \exp(-rt_i) \\
\mathbf{B} &= C_r \times [S\exp(-qT)(S^*/S)^{2\lambda}N(y_1) \\
&\quad -K\exp(-rT)(S^*/S)^{2\lambda-2}N(y_1-\sigma\sqrt{T}) \\
&\quad -K\exp(-rT)N(-x_1+\sigma\sqrt{T})+S\exp(-qT)N(-x_1)] \\
\mathbf{C} &= -\alpha\textstyle\sum_{i=1}^{k} c_i\exp(-rt_i)[N(-x_{1i}+\sigma\sqrt{t_i})+(S^*/S)^{2\lambda-2}N(y_{1i}-\sigma\sqrt{t_i})] \\
&\text{with} \\
K &= C_p \\
C_r &= \tfrac{\alpha N}{C_p} \\
x_1 &= \tfrac{\log(S/S^*)}{\sigma\sqrt{T}}+\lambda\sigma\sqrt{T} \\
y_1 &= \tfrac{\log(S^*/S)}{\sigma\sqrt{T}}+\lambda\sigma\sqrt{T} \\
x_{1i} &= \tfrac{\log(S/S^*)}{\sigma\sqrt{t_i}}+\lambda\sigma\sqrt{t_i} \\
y_{1i} &= \tfrac{\log(S^*/S)}{\sigma\sqrt{t_i}}+\lambda\sigma\sqrt{t_i} \\
\lambda &= \tfrac{r-q+\sigma^2/2}{\sigma^2}
\end{aligned}
\tag{22}
$$

The corporate bond is also priced in (A) using a risk free rate. Equation (B) prices the knock-in forward as a knock-in call minus a knock-in put. Each of the equations in Equation 22 can be found in the paper of Mark Rubinstein and Eric Reiner [22].

3.3.4 Calculation Example

Question

Starting from Equation 22 we can now price a 5 year contingent convertible with face value \$1000 on a share with a current price of \$100 and a volatility equal to 30%. The risk-free continuous interest rate is 2% and the share distributes no dividends . The conversion ($\alpha = 0.75$) will be triggered as soon the share price drops below \$35. The conversion price is equal to the price at the issue date. This sets $C_r = 7.5 = \frac{\alpha 1000}{100}$. The bank wants to issue the contingent convertible at par. What should be the coupon rate offered to the investors in order to have an initial price equal to \$1000?

Answer

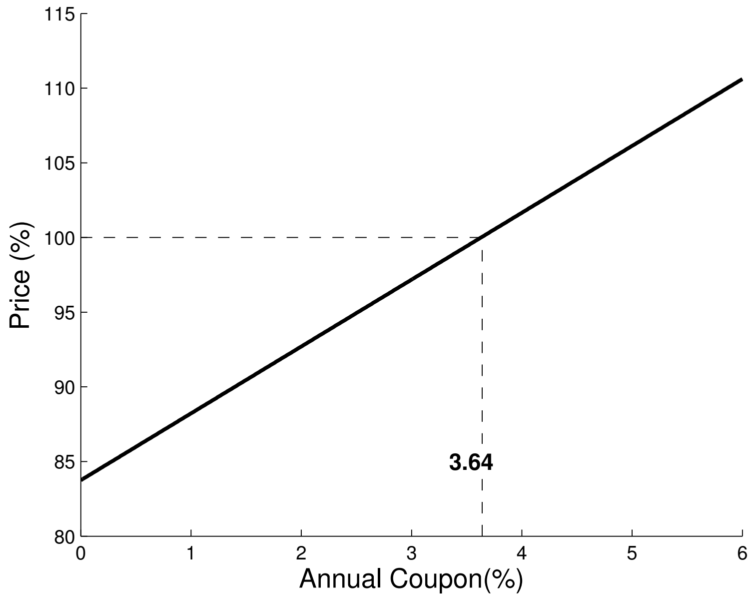

Figure 6: Price of a contingent convertible for different annual coupon levels

In Figure 6 the price of the Coco has been plotted for different coupon levels. The theoretical price for this contingent convertible offering a coupon of 3.64% is par. Using the knock-in forward model, this is the annual coupon which corresponds to a price=$1000. Given the conversion price of $100 and a partial conversion with $\alpha = 0.75$, each bond would be converted into 7.5 shares.

Calculations

Taking the coupon of 3.64%, we can workout in detail the calculations. The value of the Coco can be broken down in several components:

- **Long Corporate bond=107.63%** (Part A of Equation 22)
  The table below lists all the cash flows and the corresponding discount factors (DF) adding up to a total present value (PV) of 1076.31 :

T	Coupon	N	DF	PV
1	36.4		0.9802	35.68
2	36.4		0.9608	34.97
3	36.4		0.9418	34.28
4	36.4		0.9231	33.60
5	36.4	1000	0.9048	937.77
			Sum	**1,076.31**

- **Long Down-and-In forwards = -6.74%** (Part B of Equation 22)
There are 7.5=($\alpha N/C_p$) down-and-in forwards embedded in the structure. The pricing parameters are : $S = 100, r = 0.02, q = 0, \sigma = .3, \alpha = 0.75, S^* = 35, C_p = 100$. The price for one such a knock-in is -\$8.98. The total value of this option package in the contingent capital is :

$$-7.5 \times \frac{8.98}{1000} = -6.74\%$$

The probability of possibly having to face a situation where a trigger materializes and which converts the bond into shares, obviously reduces the value of the bond.

- **Short Binary down-and-in barrier options =-0.87%** (Part C of Equation 22)
There are in total 5 binary barrier options embedded in the structure, one maturing on every coupon date. Each of the annual coupons is worth \$36.4 and can be knocked out if before the coupon date if the trigger is breached. This corresponds to a short down-and-in barrier option (BDI). The rebate of each BDI is \$36.4.

T	**BDI**
1	0.022
2	0.621
3	1.974
4	3.571
5	5.124
	11.311

The short position in the binary down and in options, with a rebate equal to the coupon has a total value of \$11.311. Because the full face value of this sample coco is not converted, the value embedded within the Coco is equal to :

$$-0.75 \times \frac{11.311}{1000} = -0.85\%$$

Adding the value of the three components together results to a total theoretical value of the contingent convertible equal to 100.04%. The annual coupon of 3.64% provides a yield pick up above the continuous risk free interest rate. This is the compensation for the downside risk within the contingent bond.

3.3.5 Case Study: Lloyds Banking Group

The derivatives approach can easily be applied on one of the LLoyds Contingent Convertibles. We limit ourselves to one example which corresponds to the largest issue of the different ECN series issued by Lloyds. This is a CoCo denominated in GBP with an annual coupon of 15%. Of this particular bond (ISIN:XS045089255), the total amount issued is more than \$1bn. The conversion price is 59 pence, while the current price of Lloyds is 60.75 pence. The pricing of this particular case study was done on March 21, 2011. More specific bond details can be found in the following Table :

PRICING DATE	21-March-11
ISIN	XS0459089255
MATURITY	12/21/2019
PRICE	135.529
ACCRUED	2.735
DIRTY PRICE	138.264
CURRENCY	GBP
COUPON	15%
FREQUENCY	Semi-Annual
FACE VALUE	1000

The pricing parameters we considered when applying the equity derivatives approach on the valuation of this Coco :

Parameter	**Value**
r	3.42%
q	0%
σ	39%
S	0.6075
C_p	0.5900
C_r	1,695
Date	March 21, 2011

The Lloyds Cocos are constructed with a Tier 1 trigger set at 5%. The derivatives model however is developed using a market trigger where the share price S, sets off the conversion. Similar to the credit derivatives method, we can associate an implied market trigger S^* with this accounting trigger. Assume for example that the share price S^* corresponding to such a the Core Tier 1 trigger, is equal to 35 pence. This is 57.61% of the closing share price on the pricing day. The Coco is equal to a corporate bond worth GBP 1890.60 to which the knock-in forward and the binary barrier options have to be added. Using a market trigger equal to 35 pence, the value of a long position in one down-and-in forward using this particular barrier, is equal to GBP -0.085. Given the fact that the conversion ratio $C_r = 1695$, the total value of the down-and-in is GBP -144.03 (= -1695 X 0.085). For each of the coupons the value of the corresponding barrier options (BDI) is given in the following Table :

NBR	DATE	CASHFLOW	BDI
1	7/21/11	75	1.243
2	1/21/12	75	10.350
3	7/21/12	75	18.366
4	1/21/13	75	24.224
5	7/21/13	75	28.406
6	1/21/14	75	31.553
7	7/21/14	75	33.882
8	1/21/15	75	35.682
9	7/21/15	75	37.033
10	1/21/16	75	38.080
11	7/21/16	75	38.860
12	1/21/17	75	39.442
13	7/21/17	75	39.852
14	1/21/18	75	40.132
15	7/21/18	75	40.298
16	1/21/19	75	40.371
17	7/21/19	75	40.366
18	12/21/19	62.3	33.485
		Sum	**571.626**

The sum of the binary knock-in options (BDI) is equal to GBP 571.63. This value needs to be subtracted from the corporate bond and the knock-in forward. The theoretical value of this particular Lloyds Coco if the trigger was set at a share price equal to 35 pence, is equal to GBP 1174.94. This is the sum of the different components :

1. Corporate Bond : **1890.60**
2. Down-and-in Forward : **-144.03**
3. Down-and-in Barrier Options : **-571.63**

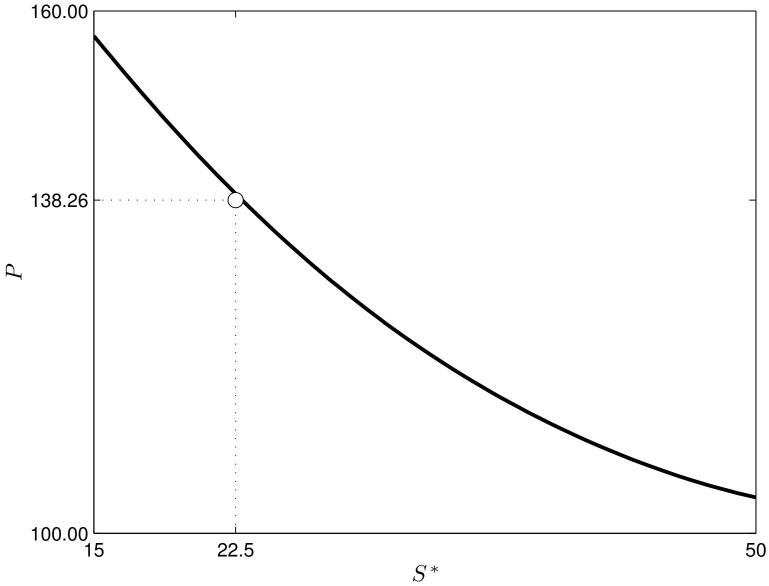

Figure 7: For different expected trigger levels S^*, the corresponding CoCo prices for LLoyds have been calculated. The implied trigger level is 22.5 pence

The three components add up to a total price equal to 117.49% of the face value. The market price of the Coco on March 21, 2011 is higher. The dirty price of this contingent convertible is 138.264%. From this market price, we can already conclude that the implied trigger level is going to be lower than 35 pence. In Figure 7 the theoretical value of the contingent convertible has been calculated for different levels of S^*. The current market price P corresponds to a trigger level of 22.5 pence. This is 37% of the closing price of LLoyds on March 21, 2011.

4 Dynamics

4.1 Introduction

Once issued, the contingent convertible will inherit all the characteristics of standard corporate debt and much more. Portfolio managers will apply without any doubt traditional risk measures such as modified duration or yield to maturity, to put this instrument in a fixed income perspective. As long as the risk of being converted into shares is very low, there is no harm in this approach. This corresponds to a situation where all is bright for the issuing bank. The bank is producing healthy profit numbers, has liquid assets and its common equity base is

more than strong enough to weather any storm. The risk of the national regulator making a bold move over a weekend and declaring the bank non-viable is very low ($\lambda_{\textbf{Trigger}} \approx 0$).
More sophisticated investors could have set up a possible hedge. A short position in the share of the bank for example would offset to a certain extend the losses on the CoCo. This is the most challenging moment in the life of the CoCo and is mind-boggling to the investor. Some would argue that exactly this delta hedging will cause the CoCo to trigger. Along the same lines CoCo opponents will claim that the mere existence of a contingent convertible will make the share price of the bank prone to become a victim of speculation and manipulation. Having an important overhang of contingent convertible debt on the balance sheet, will also impact the way the bank is managed. Especially close to the trigger, it remains to be seen how the bank's executives are going to deal with this situation. Will they try avoid at any price a conversion and protect their existing share holders from a dilution? Or, will they act as double or nothing poker stars, for whom a full conversion is the only way out of the crisis? The price behavior of a CoCo is tied closely to the design of this instrument. More in particular, the choice of the conversion price mechanism is important

4.2 Equity Sensitivity (Δ)

For illustration purposes, we can fall back on the option pricing model based on knock-in forwards. Consider a sample contingent convertible where the underlying share price is worth \$100. Using a volatility of 40% and a market trigger of \$35, the price of the contingent convertible changes with the time to maturity (T) and the underlying share price (S). Let's assume the share does not distribute dividends and that the conversion price of the Coco is \$100. On conversion the bondholder will therefore receive one share per converted CoCo. The sensitivity of the CoCo to changes in the share price is more outspoken when the remaining time to maturity is low. The delta of the contingent convertible quantifies this sensitivity of a change in the CoCo price (P) for a change in S. Close to the trigger, the delta when the remaining maturity is one year, is larger than when the CoCo has four more years outstanding.

$$\frac{\partial P}{\partial S}_{T=1} > \frac{\partial P}{\partial S}_{T=4} \tag{23}$$

On top of this, the delta on the trigger at \$35 is discontinuous. This delta is plotted in Figure 8 for our sample CoCo. It shows a weakness in the design of the contingent convertible when a market trigger such as S is used. When the CoCo is close to expiry ($T = 1$), the value of Δ can be a lot higher than 1. Once triggered the delta drops to one, because the investor receives in this case one

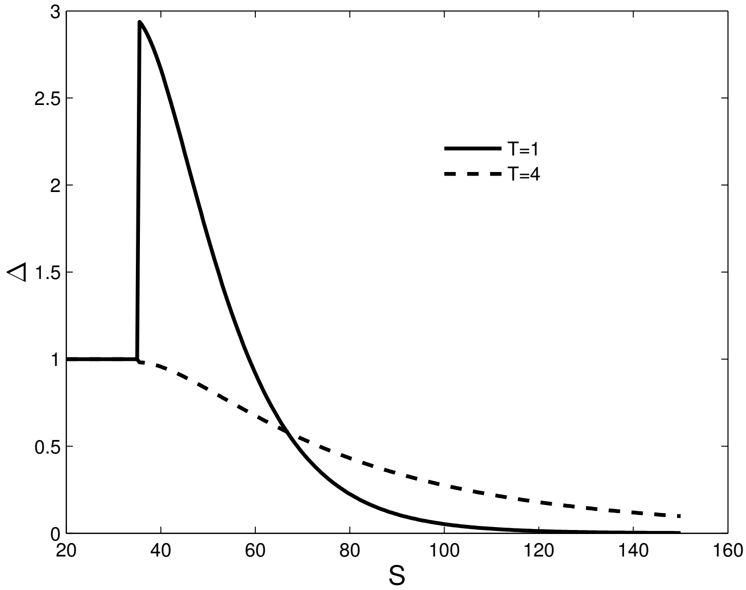

Figure 8: Delta for two different remaining times to maturity (1 and 4 years)

share per Coco. Just before the trigger materializes and with one more year to go to reach the maturity date, Δ reaches almost a value equal 3. The investor should, relying on this theoretical model, sell 3 shares for every CoCo held. As time passes by and approaches the maturity date, this number will increase even more. It is hence possible to encounter a situation, where it is exactly this hedging behavior of the investors that will trigger the conversion. The more the investors hedge their CoCos, the more the share price will go down. After which the investors will even have to sell more shares to eliminate all the equity exposure in this contingent convertible. The triggering of the contingent convertible can become a self-fulfilling prophecy, a real "death spiral". Even CoCos with an accounting or regulatory trigger are prone to this kind of behavior. Investors will sell short the shares as a hedge against the long position, they are about to receive. The trigger may be set on a particular Core Tier 1 ratio, but a collapsing share price will worry creditors and depositors regardless of the last disclosed Core Tier 1 ratio. This behavior will be outspoken when there is a lack of liquidity in the underlying shares of the CoCo.

A prudent design can avoid this situation and neutralize as much as possible any market manipulation:

- **Floor on Conversion Price**
  When the conversion price is set at the trigger moment, there is in theory a

possibility to face an unlimited dilution. The CoCo investor could receive a huge amount of shares and the delta hedging will crush the share price. A floor on this conversion price prevents this from happening to some extend. This is the solution chosen by Credit Suisse, when it issued its first Coco in February 2011.

- **Averaging on Conversion Period**
  The bank could also impose that the conversion price is not taken at the trigger moment, but as the weighted average over a pre-determined timespan preceding this event. This will smooth the delta hedging over a larger period and reduce the impact on the share price.

- **Issue size**
  There is a ceiling to the amount of contingent debt a bank can issue. The free float of the shares should be more than large enough to absorb any delta hedging activity close to the trigger event. This possibly limits the issue size of a Coco.

4.3 Gamma (Γ)

The delta hedging process described above puts the investor in the role of a villain. Hedging the CoCo could possibly lead to a situation where one is pushing down the share price and undermining the confidence of the market in the bank. This behavior could cause a run on the bank if the majority of the deposit holders lose their confidence in the bank. It is a very biased view to look at the role of the investor who is trying to hedge his bet by shorting shares. This investor is dealing with negative convexity as the CoCo approaches the trigger event :

$$\Gamma = \frac{\partial^2 P}{\partial^2 S} = \frac{\partial \Delta}{\partial S} < 0 \tag{24}$$

The investor gets more exposed to shares (Δ increases) when the share price drops and vice versa when the shares rebound from their lows. Just imagine the situation where an investor has neutralized his holdings in a contingent convertible using an appropriate delta hedge. Very soon after this hedge has been put on however, the bank announces that it was able to sell an important portion of the bad loans on its balance sheet. If this was done at no loss, it is obviously good news for the bank avoiding being triggered on its CoCos. The share price shoots up, credit spreads tighten and the level of implied volatility also responds positively to this good news. The unhedged CoCo investor joins the ranks of all those who are now delighted that the bank was able to avoid the worst to happen. On the other hand, the more sophisticated CoCo investor is now forced to buy back his

short position in the shares and is realizing a loss doing so. Running a portfolio of negative convexity in volatile and illiquid times is a challenge.
The risk exists that the share price is sucked into a terrible self-reinforcing spiral downwards. Because of this spiral, the bank would be "toast" according to CoCo-opponents. They build their case on the fact that Northern Rock was killed by a lack of liquidity not a lack of capital [4].

5 Conclusion

All of the equations we developed in this paper have their roots in a Black-Scholes setting. Because of this we came up with easy to use closed form solutions to price contingent debt. CoCos are instruments that carry a lot of fat-tail risk however. This is a typical risk that the Black-Scholes model can only handle by using a higher volatility for low strike options.
These instruments demand a stock-price model that deals with this particular kind of risk. In a next step we are going to extend our approach to more adequate processes such as Levy or Variance Gamma. The closed form solution will then have to be replaced by a simulation based model.

References

[1] Auto-callables begin to find a U.S. audience. *Derivatives Week*, XVI(1), January 2007.

[2] How to price an autocallable note with OVSN. Technical report, Bloomberg, November 2009.

[3] Final report of the commission of experts for limiting the economic risks posed by large companies. Technical report, Commission of Experts (Swiss National Bank, September 2010.

[4] Philip Aldrick. Mervyn King's plan for bank capital 'will backfire'. *The Telegraph, February 11*, 2010.

[5] Tracy Alloway. Will rate coco bonds for food. Published on : www.ft.com/alphaville, November 2010.

[6] Boris Albul andDwight M. Jaffee and Alexei Tchisty. Contingent convertible bonds and capital structure decisions. April 2010.

[7] Louise Bowman. Bank capital: Cocos could be tough nut to crack. Published on www.euromoney.com, December 2010.

[8] Michelle Brennan and Bernard de Longevialle. Potential $1 trillion bank contingent capital-style issuance faces uncertain investor interest. Technical Report Ratings Direct, Standard & Poors, December 2010.

[9] Commission of the European Communities. Directive of the european parliament and of the council on alternative investment fund managers. Technical report, April 2010.

[10] Shadow Financial Regulatory Committee. The case for a properly structured contingent capital requirement. Technical report, American Enterprise Institute (AEI), December 2010.

[11] Darrell Duffie and Kenneth J. Singleton. *Credit Risk*. Princeton Series In Finance, 2003.

[12] Mark J. Flannery. Stabilizing large financial institutions with contingent capital certificates. Technical report, Graduate School of Business Administration, University of Florida, October 2009.

[13] Jonathan Glionna and Miguel Crivelli. Analysis of bank capital. Technical report, Barclays Capital, November 2009.

[14] Richard S. Grossman. The evolution of double liability in american banking, 1865-1930. October 2002.

[15] Roberto Henriques and Saul Doctor. Making cocos work. Technical report, JP Morgan, February 2011.

[16] Frank Kennedy, Michael O'Connor, and Clodagh Muldoon. Convertible structures. Technical report, Deutsche Bank, March 2002.

[17] Radi Khasawneh and Harriet Agnew. Contingent convertibles go cococo. *eFinancial News*, March 2011.

[18] Dilip B. Madan and Wim Schoutens. Conic coconuts: the pricing of contingent capital notes using conic finance. *Mathematics and Financial Economics*, to appear.

[19] Basel Committee on Banking Supervision. Proposal to ensure the loss absorbency of regulatory capital at the point of non-viability. *Consultative Document*, August 2010.

[20] Squam Lake Working Group on Financial Regulation. An expedited resolution mechanism for distressed financial firms: Regulatory hybrid securities, April 2009.

[21] George Pennacchi. A structural model of contingent bank capital, April 2010.

[22] Mark Rubinstein and Eric Reiner. Unscrambling the binary code. *Risk Magazine*, 4:75–83, October 1991.

[23] Jan De Spiegeleer and Wim Schoutens. *The Handbook of Convertible Bonds: Pricing, Strategies and Risk Management*. Wiley, 2010.

[24] Jan De Spiegeleer and Wim Schoutens. *Contingent Convertible (CoCo) Notes: Structuring & Pricing*. EuroMoney Books, April 2011.

[25] Lujing Su and Marc Olivier Rieger. How likely is it to hit a barrier? Theoretical and emperical estimates. Technical Report Working Paper No. 594, National Centre of Competence in Research, Financial Valuation and Risk Management, October 2009.

[26] Jill Treanor. City uneasy over proposed 'coco' bonuses for barclays executives. *The Guardian*, March 2011.

[27] Emma Trincal. Volume stays strong at $410 million; two large autocallable notes top issuance. *Structured Product News*, April 2010.

[28] Derek De Vries and Marc Brehm. Coco issuance confirmed. Technical report, Bank of America Merrill Lynch, February 2011.